Stochastic volatility

Oleg Makhnin
New Mexico Tech
Dept. of Mathematics

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Model for stock price

Price of the stock at time t is X_t governed by *geometric Brownian motion*

$$d(log X_t) = \mu dt + \sigma dB_t$$

Here μdt is the deterministic component and σdB_t is the stochastic component.

B_t is Brownian motion:
dB_t is Normal with mean 0 and st.dev. $\sqrt{dt}$.

Volatility σ determines the amplitude of the process fluctuations.

Other applications: commodities, foreign currency exchange rate etc.

Applications

Volatility forecasting: important for portfolio management, risk assessment etc

Black-Scholes: A formula to obtain an Option price

Call Option is a contract to buy a security at time T (= maturity time) for the price S (=strike price).

Value of the call option:

$$C(X_t, t, T, S) = \Phi(d_1)X_t - \Phi(d_2)Se^{-r(T-t)}$$

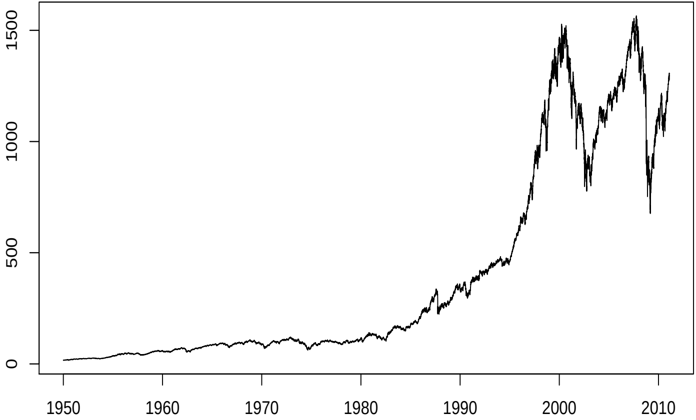
where

$$d_{1,2} = \frac{\log(X_t/S) + (r \pm \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

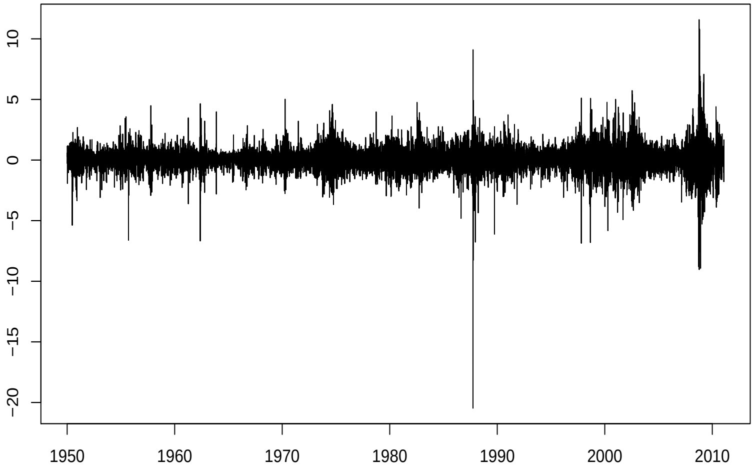
r is the risk-free rate (at which you can borrow money) and Φ is the Normal CDF.

Evidence of non-constant volatility

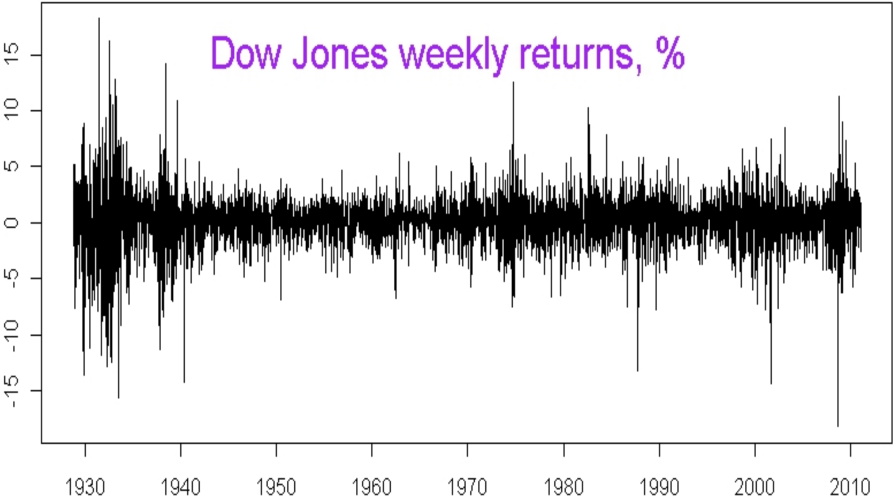
S&P500 index



S&P500 index daily returns $Y_t = (X_t - X_{t-1})/X_t$, in %



Evidence of non-constant volatility



State-space model

- $Y_t = \beta exp(h_t/2)\varepsilon_t$
- $h_t = rh_{t-1} + \sigma_\eta \eta_t, \qquad t = 2, ..., T$

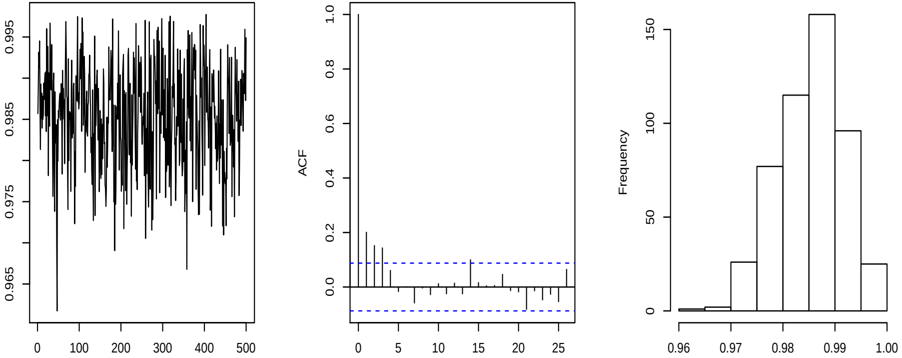
Hidden state h_t is *log volatility* and it follows an autoregressive model (r = correlation between today's state and yesterday's state).

ε_t and η_t are uncorrelated, standard Normal shocks

Efficient estimation methods since late 90's: Markov Chain Monte Carlo. Alternates between simulating from distribution of h_t and other unknown parameters (β, r, σ_η). Represents distribution of $log(Y_t^2)$ (which is not Normal) as a mixture of Normals.

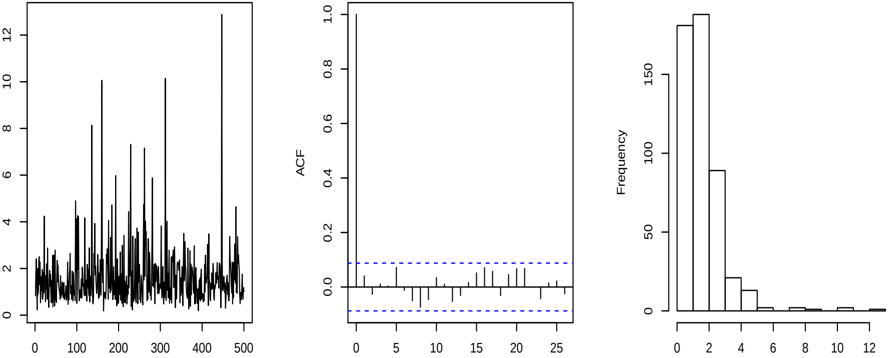
Estimation results

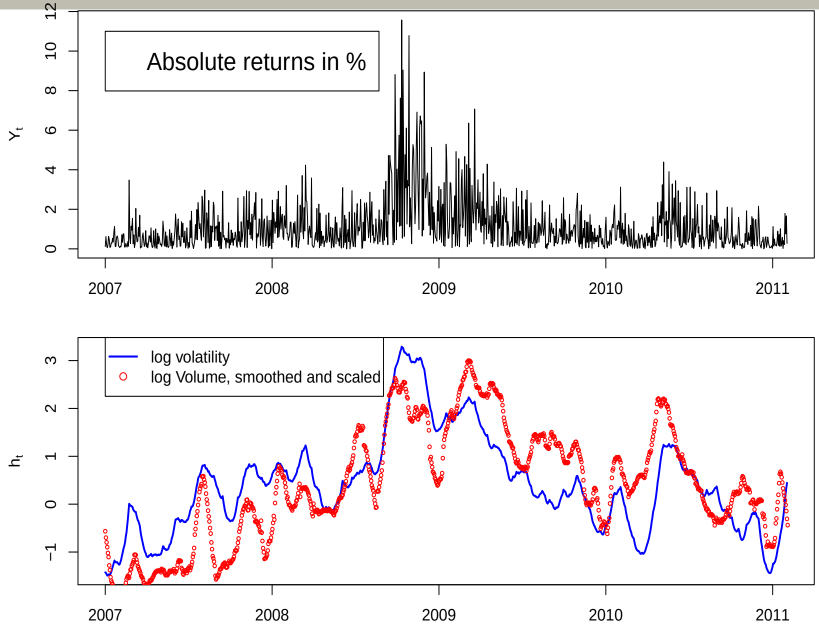
MCMC output for parameters: r (autocorrelation)



Estimation results

MCMC output for parameters: β (mean absolute return)



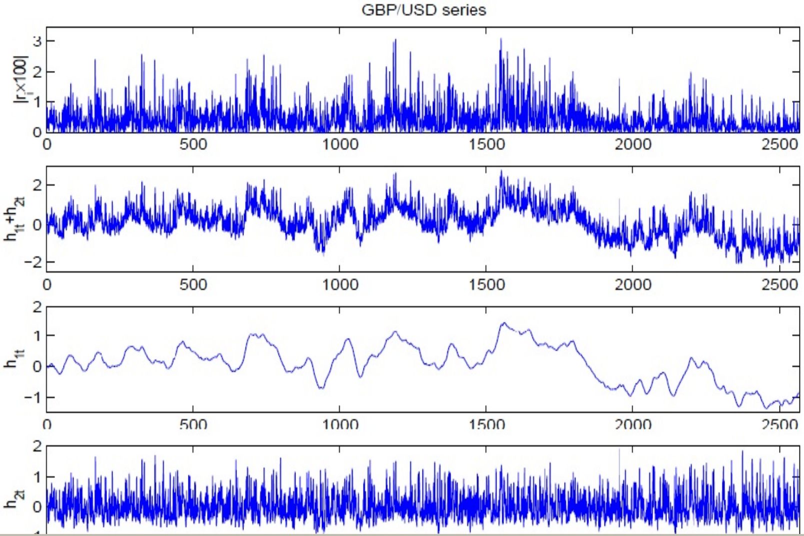

Absolute returns in %
Y_t
0
2
4
6
8
10
12
2007
2008
2009
2010
2011
log volatility
log Volume, smoothed and scaled
h_t
−1
0
1
2
3
2007
2008
2009
2010
2011

Extensions

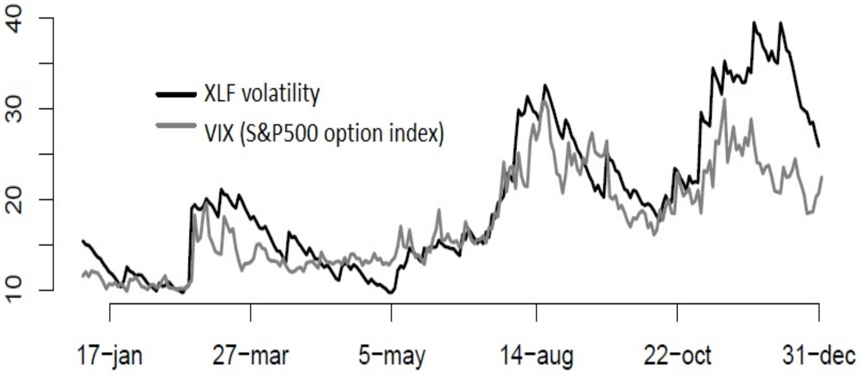
Rich, exponentially growing literature, including:

- Continuous/intraday trading
- Non-symmetry of returns
- Non-normal increments
- Models with jumps
- Multivariate: looking at several stocks at once
- Dimension reduction: hidden factor models (a few factors to explain SV behavior of several stocks)

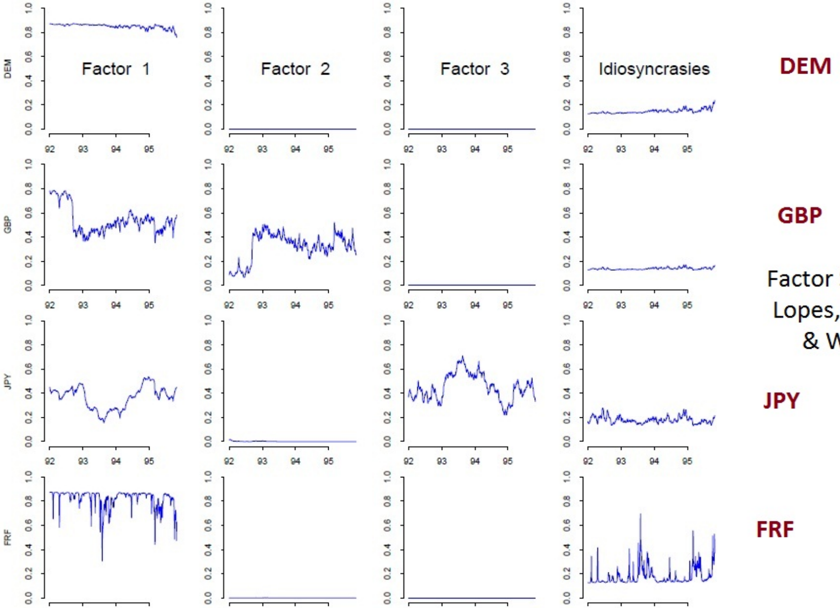
Multiscale model: Molina, Han & Fouque



Implied volatility, jumps: Lopes & Polson



2007 data. XLF: financial stocks index. VIX: volatility implied from VIX index



Factor SV:
Lopes, Aguilar & West

Possible research:

- Hidden factor models:
    - how do volatilities from different sources correlate?
    - e.g. entire market $\rightarrow$ industries $\rightarrow$ individual stocks
    - factor models: several common causes that drive the behavior of many stocks (under-explored?)
- Predictability (maybe use multiscale models)
- Development of particle-filtering methods (active area of research)
- Use in trading

Bibliography

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- Lopes & Polson *Bayesian Inference for S.V. modeling,* in Bocker, K. (Ed.) *Rethinking Risk Measurement and Reporting: Uncertainty, Bayesian Analysis and Expert Judgement*, 2010, 515-551

- Lopes & Polson *Extracting S&P500 and NASDAQ volatility: The Credit Crisis of 2007-2008* in *The Handbook of Applied Bayesian Analysis*, Oxford 2009

QUESTIONS?

THANK YOU!

see www.nmt.edu/~olegm/talks/SV