Bayesian fitting of a directional random field with application to precipitation modeling

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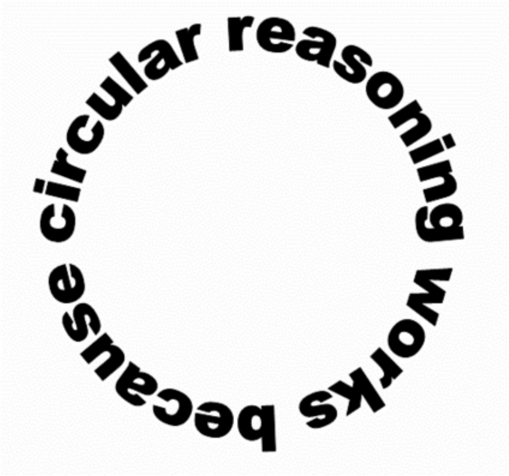
circular reasoning works because

Intro

A model for a random field W that is "pure direction"
i.e. values between 0 and 2π indexed by space.

On a regular grid: values W_i with a "smooth" prior distribution

$$p(\mathbf{W}) \propto \gamma \sum_{i \sim j} \cos(W_i - W_j)$$

where $i \sim j$ denotes neighboring grid points. Smoothness parameter γ.

Convolution method

- To interpolate values of W use well-known *convolution method*:
  For a random field X, the value of X at an off-grid location $\mathbf{y}$ is

$$X(\mathbf{y}) = \sum_{j \in \mathtt{grid}} \mathcal{K}(\mathbf{y} - \mathbf{y}_j) X(\mathbf{y}_j) \equiv K(\mathbf{y})\mathbf{X}$$

  \- a convolution of grid values $\mathbf{X}$ where $\mathcal{K}$ is some kernel function (say, Gaussian with specified width).

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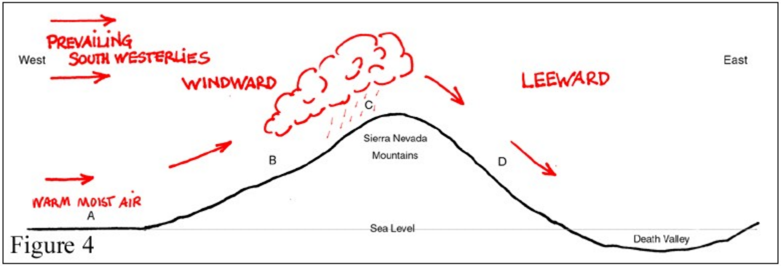
- For the circular field W, convolution is done on a circular basis

$$\cos W(\mathbf{y}) = K \cos \mathbf{W}, \quad \sin W(\mathbf{y}) = K \sin \mathbf{W}$$

Moisture flux direction

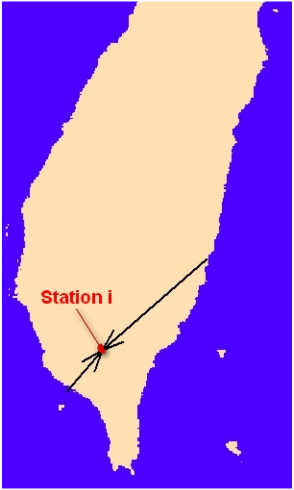
The circular field **W** describes moisture flux direction [Guan et al, 2005 did the constant W over the region]

• Interacts with terrain to produce higher precip values if terrain aspect is parallel to where the moisture comes from (windward slope) and lower on the lee slope.

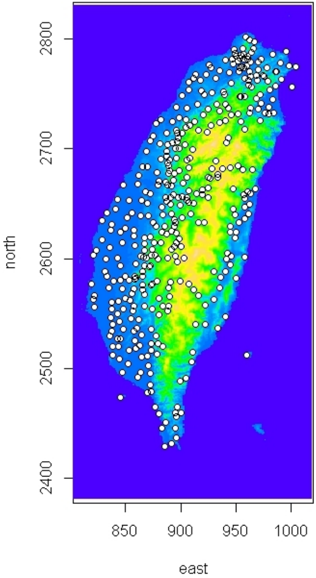


pbs.org

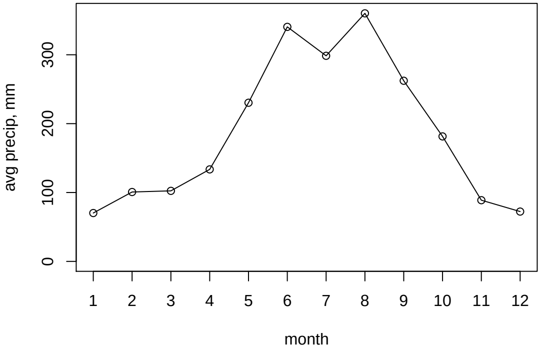
- W_i will also be used to calculate effective *distance to coast*.



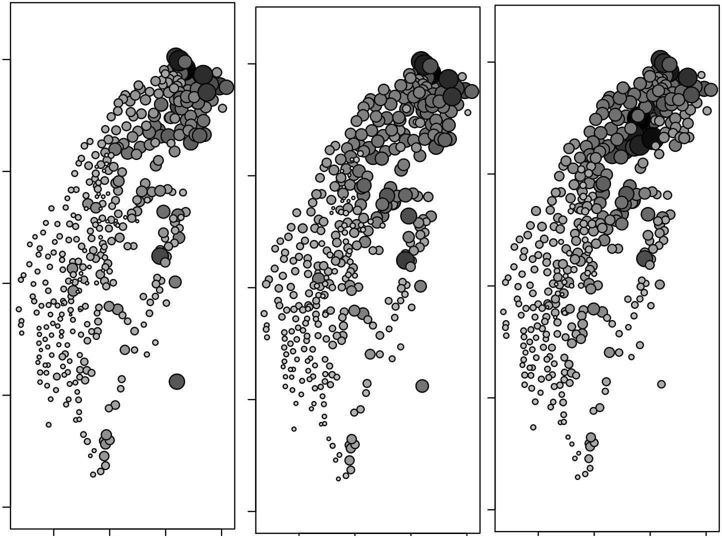
$\mathcal{D}_i(W_i)$



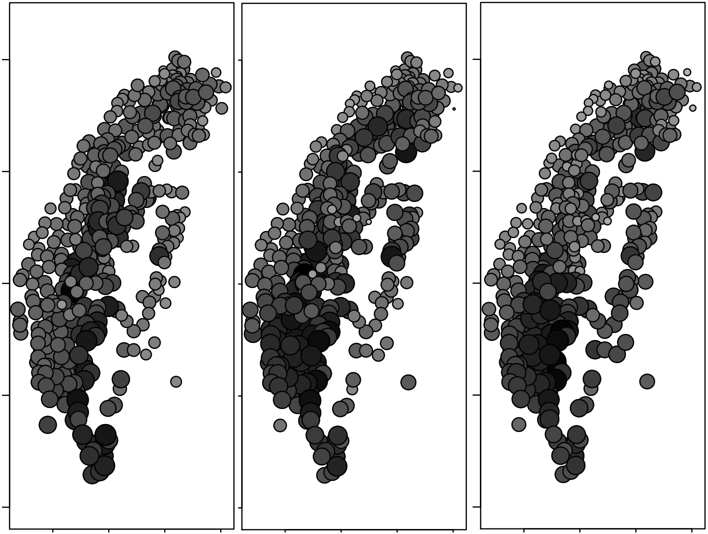
A description of Taiwan precip



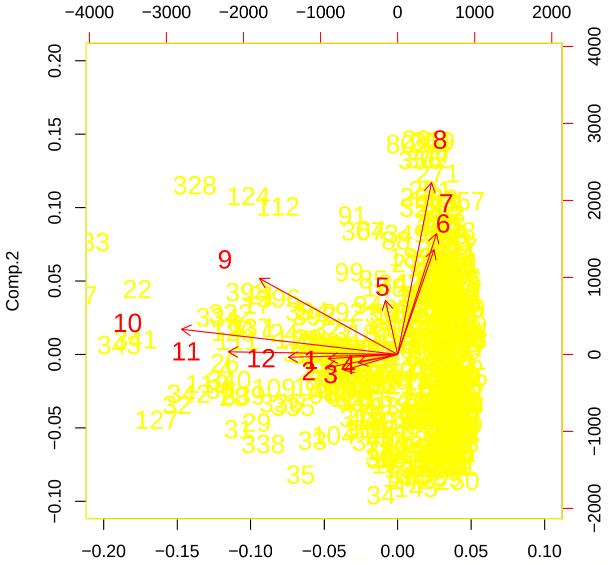
average precip January - March



average precip June-August



Principal components



The model

Let R_i = log average monthly precip for a given month, Station i

$$R_i = b_0 + b_1 Z_i + b_2 \mathcal{D}_i(W_i) + b_3 \cos(W_i - A_i) + X_i + \varepsilon_i$$

where

- X_i is a smooth local effects RF, modeled via convolution approach
- Z_i is elevation
- $\mathcal{D}_i(W_i)$ is the distance to coast in the direction of W_i
- A_i is the *terrain aspect* at Station i
- ε_i is pure nugget (white noise)

Fitting the model

Use Markov Chain Monte Carlo, with Gibbs sampler

Needs full conditional posteriors (FCPs) for:

- • random fields **W**, **X**. Use Metropolis algorithm for **W** and Gaussian FCP for **X**
- • standard methods for regression parameters $b_0, ..., b_3$ and error variance
- • due to aliasing (opposite direction of **W** and changing the sign of b_3), b_3 is restricted to be positive

Posterior quantiles of b_2, b_3

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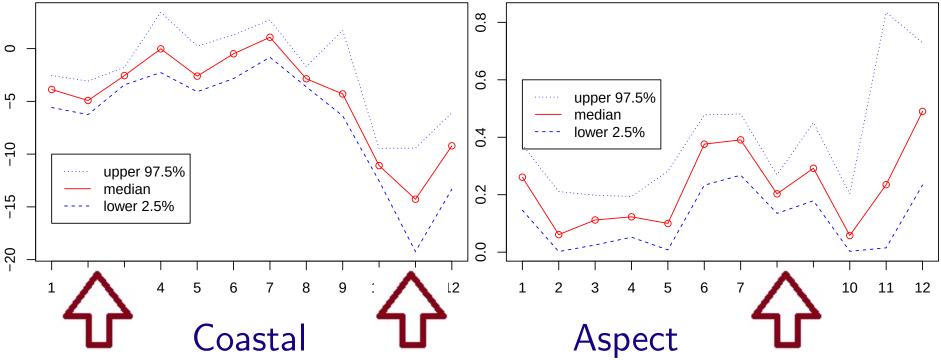
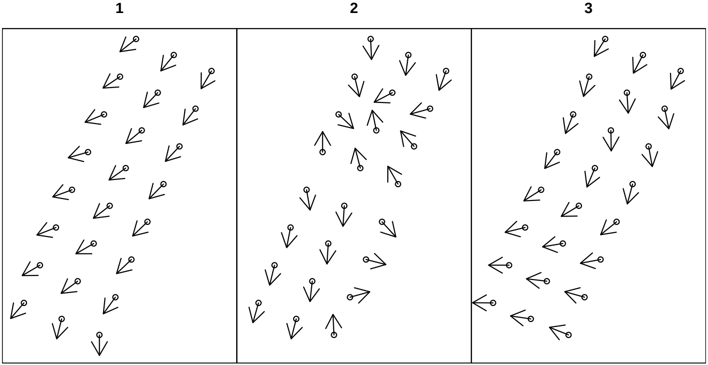
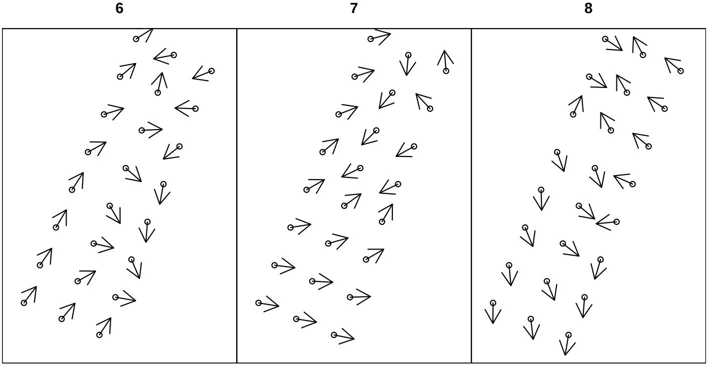


Estimates of W: January to March



Estimates of W: June to August



Conclusions

The moisture flux random field method presented:

- Allows to potentially improve the interpolation accuracy for precipitation in mountainous coastal regions

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Future work

Cross-validation study

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Future work

- Cross-validation study
- Spatially varied regression

References

Guan H, JL Wilson, O Makhnin (2005) *Geostatistical Mapping of Mountain Precipitation Incorporating Autosearched Effects of Terrain and Climatic Characteristics*, J. Hydrometeor., **6**, 1018-1031

THANK YOU!