Midterm Exam
Math 586 Fall 2011

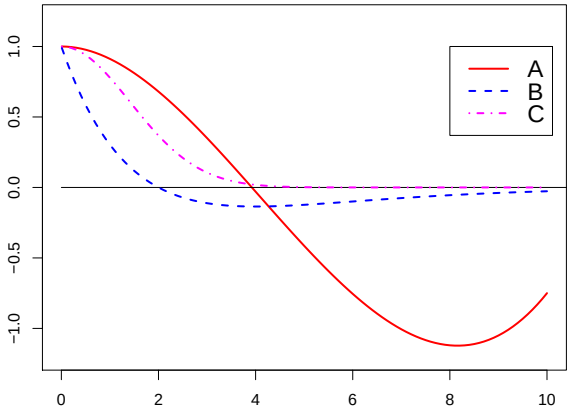
Problem	1	2	3	4	5	6	7	Total	Grade
Earned									
Possible	7	7	6	7	7	8	8	50	

1. For two observations, X_1 and X_2, with variance 1 each, denote their average as $\overline{X}$. Find $Var(\overline{X})$ when

(a) X_1 and X_2 are independent of each other

(b) Correlation coefficient r between X_1 and X_2 is 0.8

2. (a) Which of the following graphs represent allowable **covariance** models? Explain.

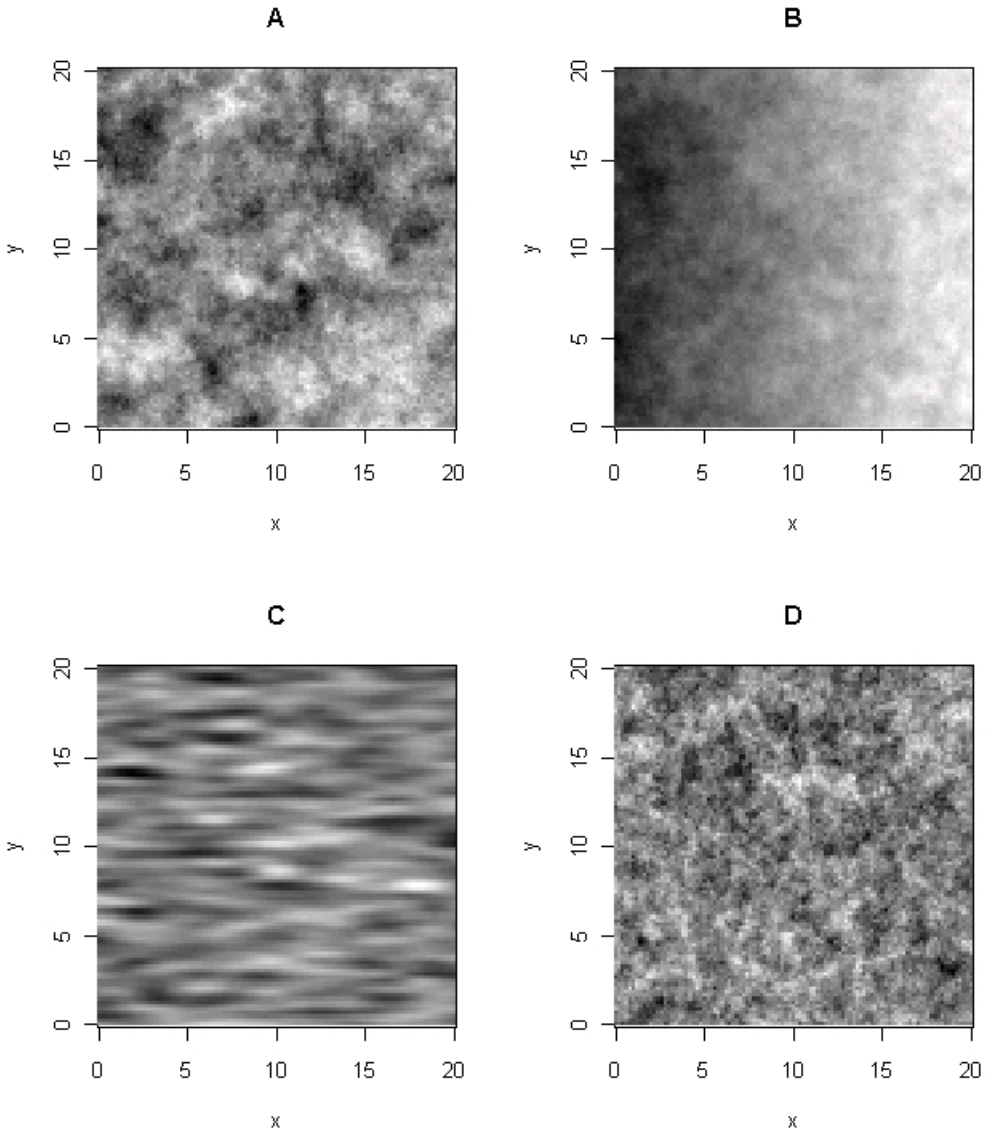


(b) Sketch an example of each

(i) a variogram model without nugget, with sill of 1 and practical range of 2.

(ii) a variogram model with a nugget and no sill

3. Which of the following 2-dimensional random fields (the value of V is given by grayscale intensity)



(a) has Gaussian variogram

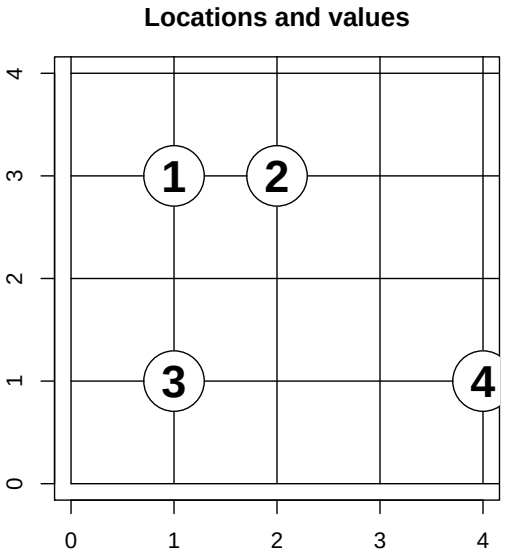
(b) Is not stationary

(c) Is not isotropic

4. (a) Explain how in Simple Kriging, the unbiasedness condition is enforced.

(b) Give an example of a process that has a variogram with no sill. Describe the way the process is constructed.

5. For the data (locations and values shown) plot the variogram cloud and compute the empirical variogram for distance classes (bins) $(0, 2.5]$ and $(2.5, 5]$



6. The vector $\mathbf{X} = (X_1, X_2, X_3)'$ has multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix

$$\mathbf{\Sigma} = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

(a) Compute the *best linear unbiased estimate* (BLUE) $\hat{X}_1$ of X_1 given X_2, X_3. That is, find the constants a_2, a_3 such that

$$\hat{X}_1 = a_2 X_2 + a_3 X_3$$

Also, find the mean square error (MSE) of this estimate.

(b) If, instead, we have found the BLUE $\tilde{X}_1$ of X_1 given X_2 only, will its MSE be higher or lower, compared to the part (a)? Explain.

7. (a) For the linear regression problem below, estimate coefficients β in the equation

$$\hat{y} = \beta_0 + \beta_1 x$$

x	y
–2	–1
–1	–1
0	0
1	1
2	1

(b) The standard deviation of residuals is 0.32 and the standard deviation of y is 1. Compute the correlation coefficient between x and y.