Math 586. Homework 1
Due Sept. 7 by 11pm

Note: if you are using computer, please attach or e-mail your code.

1. $Y = \ln K$, the natural logarithm of permeability, is assumed to be a normal random variable for a particular formation. If $\mathbb{E}[Y] = -4$, is the mean and $Var(Y) = 1.96$, is the variance, find[1]

(a) The probability that Y exceeds -5.

(b) The probability that Y is between -6 and -3.

(c) The 3rd percentile of K values (note: $K = e^Y$).

2. A simple model relates the percentage of petroleum recovery, R, in a given region to X, the porosity; Y, the permeability; and Z, the formation depth:

$$R = 0.7X + 800Y - 0.01Z$$

Random variable	X	Y	Z
Mean	40	0.02	75
St.Dev.	5	0.001	3

Find the expected value and standard deviation of R, assume that X, Y, Z are independent.

3. Mark each of the following as

i. Always true
ii. True if the random variables are independent
iii. Generally not true

(a) $\mathbb{E}[XYZ] = \mathbb{E}X\,\mathbb{E}Y\,\mathbb{E}Z$

(b) $\mathbb{E}[8X + 5Y + 4W] = 8\mathbb{E}X + 5\mathbb{E}Y + 4\mathbb{E}W$

(c) $\mathbb{E}[X^2/Y^2] = \mathbb{E}[X^2]/\mathbb{E}[Y^2]$

(d) $Var(X + Y) = Var(X) + Var(Y)$

(e) $Var(X + 2Y) = Var(X) + 2Var(Y)$

(f) $\mathbb{E}[e^X] = e^{\mathbb{E}X}$

[1] You may use Matlab's `normcdf` function, or `pnorm` function in `R`

4. Calculate the variance of the sample mean $\overline{X}$ based on the sample of identically distributed $X_1, X_2, ..., X_n$, with $\sigma^2 = Var(X_i) = 1$, when

(a) $n = 5$, all X_i are independent

(b) $n = 50$, all X_i are independent

(c) $n = 5$, the correlation between X_i and X_j equals $0.9^{|i-j|}$, $i, j, = 1, ..., n$

(d) $n = 5$, the correlation between X_i and X_j equals $(-0.9)^{|i-j|}$