Final practice

Math 483 Fall 2019.

1. Suppose $X_1, ..., X_n \sim \texttt{Poisson}(\lambda_1)$, $Y_1, ..., Y_n \sim \texttt{Poisson}(\lambda_2)$ and $n = 25$. The sample observations yielded $\overline{X} = 7.3$ and $\overline{Y} = 5.1$.

(a) Find a 95% confidence interval for $\lambda_1 - \lambda_2$, based on Normal approximation.

(b) Based on the above CI, do you believe that $\lambda_1 = \lambda_2$?

(c) Perform the Wald test for $H_0 : \lambda_1 = \lambda_2$. Find the p-value.

2. For the Exponential (mean $= \theta$) distribution

(a) Find the MLE for θ (assume $\theta > 0$).

(b) Find the Fisher information.

(c) Find the 95% confidence interval for θ, based on a sample of size $n = 100$, and $\sum_{i=1}^{100} X_i = 4132$.

(d) Perform the Likelihood Ratio test for $H_0 : \theta = 40$

3. X_1, X_2, X_3 are identically distributed with mean 0 and variance 2. Find $Var(\overline{X})$ when:

(a) $\{X_i\}$ are independent

(b) $corr(X_i, X_j) = -0.5$, $i \neq j$.

4. MISC

(a) The MLE for the mean of the exponential distribution was 2.35. Find the MLE for the median of this distribution.

(b) Are the following estimation problems regular? Explain.
i. $X_1, ..., X_n \sim^{i.i.d.}$ $\texttt{Uniform}[-\theta, \theta]$

ii. $X_1, ..., X_n \sim^{i.i.d.}$ $\texttt{Exponential}(\theta)$

(c) Find the MLE for Uniform$[-\theta, \theta]$

5. Bayes

(a) Suppose $X \sim \mathcal{N}(\theta, \texttt{Var} = 2)$, and the prior for θ is $\mathcal{N}(0, \texttt{Var} = 5)$. Find the posterior distribution for θ. Find numerical values for $X = 1$.

(b) Given $X \sim \texttt{Binomial}(n = 10, p)$ and the prior distribution for p equals $\texttt{Uniform}[0, 1]$, what is the posterior distribution? Find the posterior mean in case $X = 3$.

(c) Suppose that $X \sim \texttt{Exponential}(\beta = 1/\theta)$ and the prior $f(\theta) \propto 1/\theta$, for $\theta > 0$. Find the posterior distribution of θ. Find the posterior mean.

6. Consistent, unbiased, asymptotically Normal and plug-in estimates.

7. Highlights from Exams 1 and 2.