Hw 9 key

(10.14) (assuming μ is known)

log Lkhd $\ell(\sigma) = -n\ln\sigma - \dfrac{\sum(X_i-\mu)^2}{2\sigma^2}$

$\hat\sigma^2_{MLE} = \dfrac{\sum(X_i-\mu)^2}{n} \Rightarrow \ell(\hat\sigma_{MLE}) = -n\ln(\hat\sigma) - \dfrac{\sum(X_i-\mu)^2}{2\sum(X_i-\mu)^2/n}$

$= -\dfrac{n}{2}\ln\left(\dfrac{\sum(X_i-\mu)^2}{n}\right) - \dfrac{n}{2}$

$$\lambda = 2\ln\frac{L(\hat\sigma)}{L(\sigma_0)} = -n\left[\ln\left(\frac{\sum(X_i-\mu)^2}{n}\right) - 2\ln(\sigma_0) + 1 - \frac{\sum(X_i-\mu)^2}{\sigma_0^2}\right]$$

Wald test: $W = \dfrac{\hat\sigma_{MLE} - \sigma_0}{\hat{se}} \underset{\uparrow}{=} \dfrac{\sqrt{\dfrac{\sum(X_i-\mu)^2}{n}} - \sigma_0}{\hat\sigma/\sqrt{2n}} = \sqrt{2n}\left(1 - \dfrac{\sigma_0}{\hat\sigma}\right)$

by Example 9.29, p.134

to compare:

$$\lambda = 2n\left[\ln\left(\frac{\sigma_0}{\hat\sigma}\right) + \frac{1}{2}\left(\frac{\hat\sigma}{\sigma_0}\right)^2 - \frac{1}{2}\right]$$

and $W^2 = 2n\left(\dfrac{\sigma_0^2}{\hat\sigma^2} - 2\dfrac{\sigma_0}{\hat\sigma} + 1\right)$

(A.) $\hat\theta = \bar X$, $se(\bar X) = \dfrac{\sigma}{\sqrt n}$, $\hat{se} = \sqrt{\dfrac{\bar X}{n}} \Rightarrow W = \dfrac{\bar X - \theta_0}{\sqrt{\bar x/n}} =$

$= \dfrac{31.5 - 30}{\sqrt{31.5/120}} \simeq 2.93$, p-value $= 2 \times \text{pnorm}(-2.93)$

$\simeq .0034 < .02$

$\Rightarrow$ Reject H_0 at $\alpha = .02$.

98% C.I.: $31.5 \pm 2.326\sqrt{\dfrac{31.5}{120}} = 31.5 \pm 1.2$

or $[30.3, 32.7]$

$\Rightarrow$ also Reject H_0 because $\theta_0 \notin$ C.I.

(B.) From <u>Thm 10.6</u>

$$\beta(\theta) = 1 - \Phi\left(\frac{\theta_0 - \theta}{\hat{se}_0} + z_{\alpha/2}\right) + \Phi\left(\frac{\theta_0 - \theta}{\hat{se}_0} - z_{\alpha/2}\right)$$

will do se_0 instead of $\hat{se}$

$$= 1 - \Phi\left(\frac{0.5 - 0.55}{\sqrt{0.5(1-0.5)/500}} + 1.645\right) + \Phi\left(\frac{0.5 - 0.55}{\sqrt{\cdots}} - 1.645\right)$$

$$\simeq 0.723$$

see Hw9key.r for the plot

(C.) $H_0: p_1 = p_2 = p_3 = p_4 = p$; $\hat{p} = \dfrac{12 + 10 + 6 + 8}{35 + 50 + 48 + 52} \simeq 0.1946$

$\Rightarrow$ E_i are $n_i \hat{p} = [6.8,\ 9.7,\ 9.3,\ 10.1]$

(see Hw9key.r)

$$\chi^2 = \sum \frac{(E_i - X_i)^2}{E_i} = 5.6, \quad df = k - 1 = 3$$

p-value $\simeq 0.133$ $\Rightarrow$ no evidence against H_0