Hw5 key

M483

6.3 $\mathbb{E}(\hat\theta) = 2\,\mathbb{E}(\bar X_n) = 2\,\mathbb{E}(X_i) = 2\frac{\theta}{2} = \theta \Rightarrow bias = 0$

$\mathbb{V}(\hat\theta) = 4\,\mathbb{V}(\bar X_n) = \frac{4\sigma^2}{n} = \frac{4}{n}\frac{\theta^2}{12} \Rightarrow se(\hat\theta) = \frac{\theta}{\sqrt{3n}}$

$MSE = bias^2 + \mathbb{V}(\hat\theta) = \frac{\theta^2}{3n}$

A. Similar to Problem 3.3 (Hw3), $P(\underbrace{\max\{X_1,\dots,X_n\}}_{\hat\theta} \le y) =$

$= [P(X_i \le y)]^n = \left(\frac{y}{\theta}\right)^n$. Thus, $P(|\hat\theta - \theta| > \varepsilon) = P(\hat\theta < \theta - \varepsilon)$

$= \left(\frac{\theta - \varepsilon}{\theta}\right)^n \to 0$

B. Need to do the angles in radians, otherwise the derivative will be wrong!

$\bar W \approx N\left(\frac{3.05\pi}{180}, \left(\frac{0.28\pi}{180}\right)^2 \frac{1}{90}\right) \Rightarrow 50 \sin \bar W \approx N\left(50\sin\left(\frac{3.05\pi}{180}\right), \frac{50^2\cos^2(\mu)}{90}\left(\frac{0.28\pi}{180}\right)^2\right)$

$[g(w) = 50\sin(w)]$, $\mu = \frac{3.05\pi}{180}$

$= N(2.66, .026^2)$

See Hw5key.r for the simulation.

C. $(\bar X - \bar Y) \pm z_{\alpha/2}\sqrt{\frac{S_x^2}{n_x} + \frac{S_y^2}{n_y}} = (24.8 - 25.6) \pm 2.~~57~~6 \cdot \sqrt{\frac{2.3^2}{80} + \frac{1.6^2}{100}}$

$\Rightarrow -.8 \pm .78$, or $[-1.58, -.02]$

D. Since S_n^2 is a consistent estimator of σ^2, for $g(\theta) = \ln(\sqrt{\theta}) = \frac{1}{2}\ln\theta$ is a continuous function $\Rightarrow$ by Thm 5.5(f) $\hat\tau = g(S_n^2)$ is a consistent estimator of $g(\sigma^2) = \ln\sigma$.

Since $\mathbb{E}(S_n^2) = \sigma^2$ (unbiased), and $g(\theta)$ is a concave function, we expect $\mathbb{E}g(\theta) < g(\mathbb{E}\theta)$, that is $\hat\tau$ should be biased low.

See Hw5key.r for a simulation.