HW4 key

M483

(4.) $X_n \xrightarrow{P} 0$, because $P(|X_n - 0| > \varepsilon) = P(X_n > \varepsilon)$

$= \frac{1}{n^2}$ (as long as $\frac{1}{n} < \varepsilon$) $\to 0$

$X_n \overset{qm}{\not\to} 0$, because $\mathbb{E}(X_n - 0)^2 = \mathbb{E}(X_n^2) = \left(\frac{1}{n}\right)^2\left(1 - \frac{1}{n^2}\right) + n^2\left(\frac{1}{n^2}\right)$

> 1, does not converge to 0.

(7.) a) $P(|X_n - 0| > \varepsilon) = P(X_n > \varepsilon) \leq P(X_n > 0) =$

$= 1 - P(X_n = 0) = 1 - e^{-1/n} \to 0$

b) $P(|nX_n - 0| > \varepsilon) = P(nX_n > \varepsilon) \leq P(X_n > 0)$,

etc. like in part (a).

(9.) $P(|X_n - X| > \varepsilon) \leq P(|X_n - X| > 0) = \frac{1}{n} \to 0$

$\Rightarrow X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{D} X$ by Thm 5.4.

$\mathbb{E}(X - X_n)^2 \geq 0^2 \cdot \left(1 - \frac{1}{n}\right) + (e^n - 1)^2 \frac{1}{n} \to \infty$,

so $X_n \overset{qm}{\not\to} X$.

(14.) $\bar{X}_n \approx N\left(\mu, \frac{\sigma^2}{n}\right)$, for $X_i \sim U[0,1]$ $\mu = \frac{1}{2}$, $\sigma^2 = \frac{1}{12}$

$\Rightarrow \bar{X}_n \approx N\left(\frac{1}{2}, \frac{1}{12n}\right)$; for $g(x) = x^2$, by Delta-method

$Y_n = g(\bar{X}_n) \approx N\left(g\left(\frac{1}{2}\right), \left[g'\left(\frac{1}{2}\right)\right]^2 \frac{1}{12n}\right) = N\left(\frac{1}{4}, \frac{1}{12n}\right)$

(A.) $\bar{X} \approx N\left(20, \text{var.} = \frac{5^2}{50}\right)$, or $Z \approx \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

a) $P(\bar{X} > 21) \approx P\left(Z > \frac{21 - 20}{5/\sqrt{50}}\right) = P(Z > 1.414) \approx .079$

b) $\underset{0.90}{\underset{\parallel}{P(-z_{\alpha/2} < Z < z_{\alpha/2})}} \approx P\left(\mu - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \bar{X} < \mu + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$;

$\Rightarrow z_{\alpha/2} = qnorm(0.95) \simeq 1.645$

interval $\quad 20 \pm 1.645 \frac{5}{\sqrt{50}}$, or $\quad 20 \pm 1.2$

E. C. (#16.) Let $X \sim$ Uniform $[-1, 1]$ and $Y = -X$.

$X_n \sim$ Uniform $[-\cancel{1} - \frac{1}{n}, 1 + \frac{1}{n}]$, $\quad Y_n = X_n$; $\quad \begin{cases} X_n \xrightarrow{D} X, \\ Y_n \xrightarrow{D} X \text{ and also} \\ Y_n \xrightarrow{D} -X = Y \end{cases}$

Then $X_n + Y_n$ has a non-trivial distribution, (~~sum of two~~ $\xrightarrow{D}$ Uniform $[-2, 2]$

But $X + Y = X - X = 0$ (point mass at 0)