Hw 2 key

M483

(4.) a)

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ \frac{x}{4}, & 0 < x < 1 \\ \frac{1}{4}, & 1 < x < 3 \\ \frac{1}{4} + \frac{3}{8}(x-3), & 3 < x < 5 \\ 1, & x \geq 5 \end{cases}$$

$\frac{3}{8}x - \frac{7}{8}$ ✓

b) $F_Y(y) = P(Y \le y) = P(X \geq \frac{1}{y}) = 1 - F_X(\frac{1}{y}) =$

$$= \begin{cases} 0, & y \le 1/5 \\ 1 - \left(\frac{3 \times 1}{8 \times y} - \frac{7}{8}\right) = \frac{15}{8} - \frac{3}{8y}, & 1/5 < y < 1/3 \\ 3/4, & 1/3 < y < 1 \\ 1 - \frac{1}{4y}, & y > 1 \end{cases}$$

$$f_Y(y) = F_Y'(y) = \begin{cases} 0, & y \le 1/5 \\ \frac{3}{8y^2}, & 1/5 < y < 1/3 \\ 0, & 1/3 < y < 1 \\ \frac{1}{4y^2}, & y > 1 \end{cases}$$

(16.)

$$P(X=k \mid X+Y=n) = \frac{P(X=k, Y=n-k)}{P(X+Y=n)} \overset{\text{indep.}}{=} \frac{e^{-\lambda}\frac{\lambda^k}{k!} e^{-\mu}\frac{\mu^{n-k}}{(n-k)!}}{e^{-(\lambda+\mu)}\frac{(\lambda+\mu)^n}{n!}}$$

(done in class)

$$= \frac{n!}{k!(n-k)!}\left(\frac{\lambda}{\lambda+\mu}\right)^k\left(\frac{\mu}{\lambda+\mu}\right)^{n-k} \Rightarrow \text{Binomial}\left(\pi = \frac{\lambda}{\lambda+\mu},\ 1-\pi = \frac{\mu}{\lambda+\mu}\right)$$

21. $P(Y \le y) = P(\max(X_1,\dots,X_n) \le y) = P(X_1 \le y, X_2 \le y, \dots, X_n \le y)$

$$\underset{\substack{\uparrow \\ \text{indep.}}}{=} [P(X_i \le y)]^n = (1 - e^{-x/\beta})^n$$

$$f_Y(y) = F'(y) = n(1 - e^{-y/\beta})^{n-1}\left(\frac{1}{\beta} e^{-y/\beta}\right)$$

A. a) $P(X>9) = P\left(Z > \frac{9-5}{\sqrt{12}}\right) = 1 - \Phi(1.155) \simeq .124$

b) $P(3<X<9) = \Phi\left(\frac{9-5}{\sqrt{12}}\right) - \Phi\left(\frac{3-5}{\sqrt{12}}\right) \simeq .594$

c) $P(Z>z) = 0.14 \Rightarrow z = \Phi^{-1}(0.86) = 1.08$

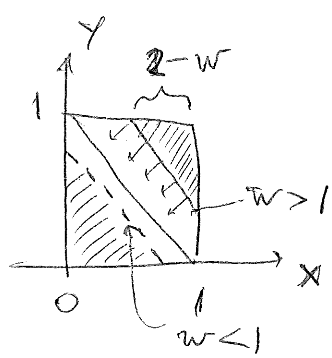
$$\Rightarrow x = 5 + \sqrt{12}(1.08) \simeq 8.74$$

d) let's find two symmetrical points a, b

First, find z: $P(-z<Z<z) = 0.88 \Rightarrow z = \Phi^{-1}\left(.5 + \frac{.88}{2}\right) \simeq 1.555$

$$\Rightarrow a, b = 5 \pm \sqrt{12}(1.555) = \begin{cases} -0.387 \\ 10.387 \end{cases}$$

B.



$$\cdot\ P(W \le w) \underset{\substack{\uparrow \\ w<1}}{=} \int_0^w \int_0^{w-y} 1\, dx\, dy = \int_0^w (w-y)\,dy$$

$$= wy - y^2/2 \Big]_0^w = \frac{w^2}{2}$$

$$\cdot\ P(W \le w) \underset{\substack{\uparrow \\ w>1}}{=} 1 - P(W > w) = 1 - \int_{2-w}^{1} \int_{w-y}^{1} dx\, dy$$

$$= 1 - \frac{(2-w)^2}{2} \quad \text{by symmetry}$$

$$\Rightarrow f_W(w) = \begin{cases} w, & 0<w<1 \\ 2-w, & 1 \le w < 2 \end{cases}$$