M483 Hwk key

(5.) $\Omega = \{HH, HTH, THH, HTTH, THTH, TTHH, \ldots\}$

$P(X=k) = p^2 q^{k-2} \binom{k-1}{1}$, with $p = \frac{1}{2}$

$P(X=k) = (k-1)\left(\frac{1}{2}\right)^k$, $k = 2, 3, \ldots$

(11.) Let $P(AB) = P(A)P(B)$. Then $P(A^c \cup B^c) = P(A^c) + P(B^c)$ $- P(A^c B^c)$

(union law)

$\Rightarrow P(A^c B^c) = P(A^c) + P(B^c) - P(A^c \cup B^c) = 1 - P(A) + 1 - P(B)$

(By de Morgan's law, $(A^c \cup B^c)^c = AB$) $-(1 - P(AB)) =$

$= 1 - P(A) - P(B) + P(A)P(B) = (1 - P(A))(1 - P(B)) = P(A^c)P(B^c)$ ✓

(12.)

G_1, G_2 can happen for this card only

$P(G_2 \mid G_1) = \frac{P(G_1 G_2)}{P(G_1)} = \frac{1/3}{1/2} = 2/3$

(15.) (a) Let X = # of children with blue eyes

$X \sim \text{Binomial}(n=3, p=1/4)$

$P(X \geq 2 \mid X \geq 1) = \frac{P(X \geq 2 \cap X \geq 1)}{P(X \geq 1)} = \frac{P(X \geq 2)}{P(X \geq 1)} =$

$= \frac{1 - (P(0) + P(1))}{1 - P(0)} = \frac{1 - (3/4)^3 - 3(3/4)^2(1/4)}{1 - (3/4)^3} = 10/37$

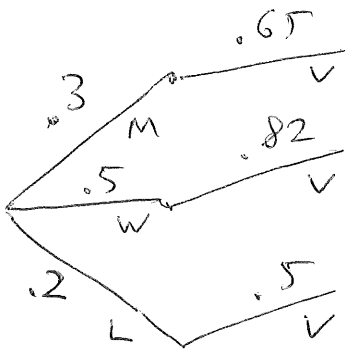
(b) $P(X \geq 2 \mid \text{youngest Blue}) = P(\text{at least one of two others Blue})$

$= 1 - (3/4)^2 = 7/16$

17. $P(A|BC)\,P(B|C)\,P(C) = P(A|BC)\,P(BC) = P(A \cap (BC)) = P(ABC)$

$P(E_1|E_2)\,P(E_2) = P(E_1, E_2)$

19.



$.3(.65)$

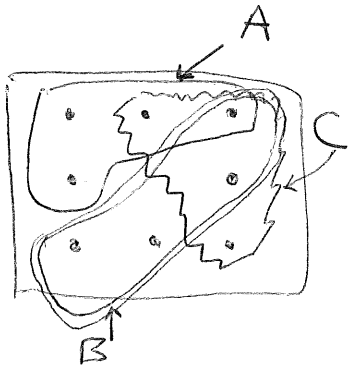
$.5(.82)$

$.2(.5)$

a) $P(V) = .3(.65) + .5(.82) + .2(.5) = .705$

b) $P(W|V) = \dfrac{.5(.82)}{.705} \simeq .5816$

E.C.



let $\#(\Omega) = 8$, equally likely.

$P(A) = P(B) = P(C) = 1/2$

and $P(ABC) = \frac{1}{8}$

but $P(AB) = \frac{1}{8} \neq P(A)P(B)$