Hw 10
key

M483

(6.) prior $f(\lambda) \propto \lambda^{\alpha-1} e^{-\lambda/\beta}$

a) lkhd $f(\vec{x}|\lambda) \propto \prod e^{-\lambda} \lambda^{x_i}$ $\Rightarrow$ posterior $f(\lambda|\vec{x}) \propto \lambda^{(\sum x_i)+\alpha-1} e^{-[\lambda/\beta + n\lambda]}$

this is gamma density with $\begin{cases} \alpha^* = \alpha + \sum x_i \\ \beta^* = \dfrac{1}{\frac{1}{\beta} + n} \end{cases}$.

posterior mean $= \alpha^* \beta^* = \dfrac{\alpha + \sum x_i}{\frac{1}{\beta} + n}$

b) $\ln f(x_1|\lambda) = -\lambda + x_1 \ln \lambda + \text{const}$

$\dfrac{\partial \ln f}{\partial \lambda} = -1 + \dfrac{x_1}{\lambda}$, $\dfrac{\partial^2 \ln f}{\partial \lambda^2} = -\dfrac{x_1}{\lambda^2}$; $I(\lambda) = -E\left(-\dfrac{x_1}{\lambda^2}\right) = \dfrac{\lambda}{\lambda^2} = \dfrac{1}{\lambda}$

$\Rightarrow$ Jeffrey's prior is $f(\lambda) \propto \dfrac{1}{\sqrt{\lambda}}$

posterior $\propto \lambda^{(\sum x_i) - \frac{1}{2}} e^{-n\lambda}$

is gamma with $\begin{cases} \alpha^{**} = (\sum x_i) + \frac{1}{2} \\ \beta^{**} = \frac{1}{n} \end{cases}$, posterior mean $= \dfrac{(\sum x_i) + \frac{1}{2}}{n}$

A. Prior $f(\theta) \propto \theta^{\alpha}$

Lkhd $f(\vec{x}\mid\theta) \propto \frac{1}{\theta^{n}}$, for $\theta > \max\{X_1,\dots,X_n\} = X_{(n)}$ (see p.125)

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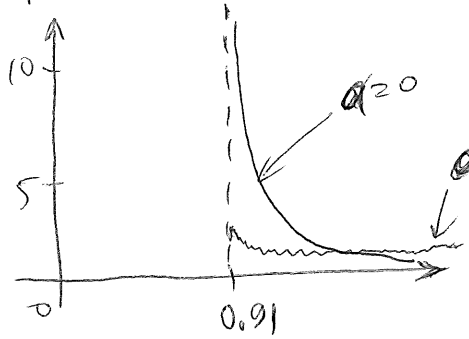
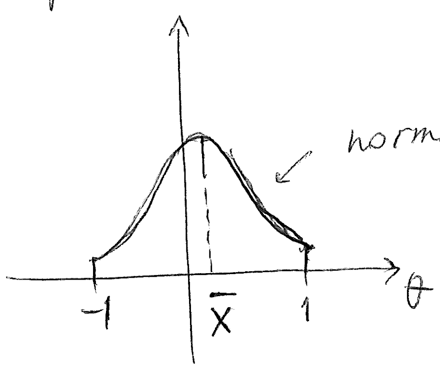
posterior $\propto \theta^{\alpha-n}$, $\theta > X_{(n)} = 0.91$



$\alpha=4$ posterior does not exist (θ^{a-n} not integrable)

B. posterior $\propto \prod\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{(X_i-\theta)^2}{2}}\right) \mathbb{1}_{\{-1\le\theta\le 1\}}$ ← prior



normal curve truncated between $-1, 1$ and rescaled to make $\int_{-1}^{1} \dots = 1$ (exact scaling depends on $\bar{X}$)

C. prior $f(\theta) \propto e^{-\alpha\theta}$

Lkhd $f(\vec{x}\mid\theta) \propto \prod\left(\theta e^{-\theta X_i}\right)$

$\Downarrow$

posterior $\propto \theta^{n} e^{-\theta(\alpha + \sum X_i)}$: it's Gamma density with $\alpha^* = n+1$, $\beta^* = \frac{1}{\alpha + \sum X_i}$

$\Rightarrow$ posterior mean $= \alpha^*\beta^* = \frac{n+1}{\alpha + \sum X_i}$