Exam 2 Name (please print) ____________________

Math 483, Fall 2017 November 21, 2017

Show work and correct notation for full credit. Give numerical or simple fraction answers whenever possible.

Problem	1	2	3	4	5	6	Total
Earned							
Possible	10	10	10	10	10	10	50

Solve 5 problems out of 6. Cross one out.

1. The success percentages in two groups are p_1 and p_2 respectively. Two groups of 80 observations each were collected. The first group had 37 successes, and second group had 24 successes.

(a) Find a 95% confidence interval for $p_1 - p_2$, based on Normal approximation.

(b) Based on the above CI, do you believe that $p_1 = p_2$?

(c) Perform the Wald test for $H_0 : p_1 = p_2$. Also, find the p-value.

2. (a) Let $\hat{\theta}$ be asymptotically Normal($\texttt{mean} = 17.1, \texttt{variance} = 4.5/n$). Use Delta method to find the approximate distribution of $\ln(\hat{\theta})$

(b) Give an example of an estimation problem where MLE is not asymptotically Normal.

3. MLE
Let the i.i.d. sample $X_1, ..., X_n$ come from the Poisson distribution with unknown mean $\theta > 0$.

(a) Find the MLE for θ. Evaluate numerically based on a sample of size $n = 100$, and $\sum_{i=1}^{100} X_i = 815$.

(b) Find the Fisher information

(c) Describe the asymptotic distribution of the MLE as n gets large.

4. For the Exponential distribution with unknown mean $\theta = \beta$ perform the likelihood ratio test for

$$\begin{cases} H_0: & \theta = 10 \\ H_1: & \theta \neq 10 \end{cases}$$

based on the data: $n = 50$, $\overline{X} = 14$.

5. A lottery can have 3 kinds of prizes, A, B and C. We'd like to test the hypothesis that Prize B and Prize C are equally likely, while Prize A is twice as likely as each of B and C. The empirical data on prizes gave the following counts:

A	B	C
38	12	16

Perform the chi-square test. Give your conclusion at the level $\alpha = 0.1$.

Table C: Critical points of the chi-square distribution

	Upper tail probability						
Degrees of freedom	0.100	0.050	0.025	0.010	0.005	0.001	0.0005
1	2.706	3.841	5.024	6.635	7.879	10.828	12.116
2	4.605	5.991	7.378	9.210	10.597	13.816	15.202
3	6.251	7.815	9.348	11.345	12.838	16.266	17.730
4	7.779	9.488	11.143	13.277	14.860	18.467	19.997
5	9.236	11.070	12.833	15.086	16.750	20.515	22.105

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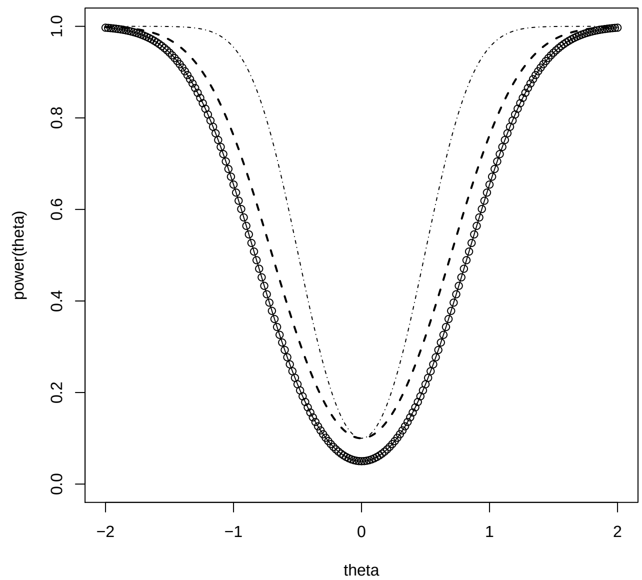
6. Misc.

(a) Match each of the power curves below with one of the following cases:

i. $n = 100, \alpha = 0.1$ ii. $n = 50, \alpha = 0.1$ iii. $n = 50, \alpha = 0.05$



(b) The MLE for the mean of the exponential distribution was 2.35. Find the MLE for the median of this distribution.

(c) True or False? Explanations are not necessary, but they won't hurt.

i. Fisher information matrix contains approximate values of variances and covariances of maximum likelihood estimates.

ii. If $\hat{\theta}$ is an MLE for θ then $\ln(\hat{\theta})$ is the MLE for $\psi = \ln(\theta)$.