## Exam 2 Name (please print) \_

Math 483, Fall 2017 November 21, 2017

Show work and correct notation for full credit. Give numerical or simple fraction answers whenever possible.

D., a la la	1	2	2	1	- E		T-4-1
Problem	1	2	3	4	)	O	Total
Earned							
Possible	10	10	10	10	10	10	50

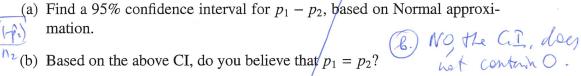
C. W= Paliner + Paliner

## Solve 5 problems out of 6. Cross one out.

a) ( ln (17.1),

 $g(\theta) = \ln(\theta)$   $g'(\theta) = \frac{1}{2}$ 

1. The success percentages in two groups are  $p_1$  and  $p_2$  respectively. Two groups of 80 observations each were collected. The first group/had 37 successes, and second group had 24 successes.



- (c) Perform the Wald test for  $H_0: p_1 = p_2$ . Also, find the p-value. Reject the problem than  $p_2$ 0.1625 ± 0.1484 ~ [,01,,31]
  - Delta method to find the approximate distribution of  $ln(\hat{\theta})$ (b) Give an example of an estimation problem where MLE is not asymptotically Normal (b) Give an example of an estimation problem where MLE is not asymptotically Normal.

    is not a regular estimation problem.  $P(H \in X) = \frac{x}{\theta}$ , does not converge to Normal dist.

    - Let the i.i.d. sample  $X_1, ..., X_n$  come from the Poisson distribution with unknown mean  $\theta > 0$ .

(a) Let  $\hat{\theta}$  be asymptotically Normal(mean =17.1, variance = 4.5/n). Use

- (a) Find the MLE for  $\theta$ . Evaluate numerically based on a sample of size
- $e'(\theta) = -n + \frac{\xi x_i}{\theta} = 0 \Rightarrow \theta = \frac{\xi x_i}{\eta} = x$ (b) Find the Fisher information
- (c) Describe the asymptotic distribution of the MLE as n gets large. (6) ln f(0/= -+ + X ln + + const,  $\hat{\theta} \approx \mathcal{N}(\theta, var = \frac{\theta}{n})$ 2 lnf = - x 2 lnf = - x 2 lnf = - x 22  $I(\theta) = -E(\frac{\partial^2 \ln t}{\partial \theta^2}) = \frac{EX}{\theta^2} = \frac{1}{\theta} = \frac{1}{\theta}$   $I_n(\theta) = \frac{n}{\theta}$

$$\mathcal{L}(\theta) = \Pi + e^{-\frac{xi}{\theta}} = \mathcal{L}(\theta) = \frac{2(-\ln \theta - \frac{xi}{\theta})}{2(-\ln \theta - \frac{xi}{\theta})}; \text{ recall that }$$

$$\chi = 2\left(\ell(\theta) - \ell(\theta_0)\right) = -2n\left[\ln x + \frac{x}{x} - \ln \theta_0 - \frac{x}{\theta_0}\right] - n\left[\ln \theta + \frac{x}{\theta}\right]$$

**4.** For the Exponential distribution with unknown mean  $\theta = \beta$  perform the likelihood ratio test for

$$\begin{cases} H_0: \ \theta = 10 \\ H_1: \ \theta \neq 10 \end{cases} = 6.35. \text{ At } \alpha = 0.05$$

based on the data: n = 50,  $\overline{X} = 14$ .

**5.** A lottery can have 3 kinds of prizes, A, B and C. We'd like to test the hypothesis that Prize B and Prize C are equally likely, while Prize A is twice as likely as each of B and C. The empirical data on prizes gave the following counts:

A B C 
$$\frac{E_{1}}{33}$$
  $\frac{A}{16.5}$   $\frac{B}{16.5}$ 

Perform the chi-square test. Give your conclusion at the level  $\alpha = 0.1$ .

Table C: Critical points of the chi-square distribution

		Upper tail probability									
	0.100	0.050	0.025	0.010	0.005	0.001	0.0005				
Degrees of											
freedom											
1	2.706	3.841	5.024	6.635	7.879	10.828	12.116				
2	4.605	5.991	7.378	9.210	10.597	13.816	15.202				
3	6.251	7.815	9.348	11.345	12.838	16.266	17.730				
4	7.779	9.488	11.143	13.277	14.860	18.467	19.997				
5	9.236	11.070	12.833	15.086	16.750	20.515	22.105				

[next page]

= (38-33) +

+ (16-16.5)2

-2.0

X. 10, df-2

[3] a) 
$$\frac{1}{2} = \frac{1}{2}$$

[3] b)  $\beta = 2.35$ ,  $\frac{1}{2} = \frac{1}{2}$ 

[3] c) i. Take (Inverse of II reeded)

[2+2] True (equivariance principle)