

Math 382

Practice Exam 2

1. Consider the PDF of a random variable X

$$f(x) = \begin{cases} c(5+x) & -5 \leq x \leq 0 \\ c(5-x) & 0 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find c that makes $f(x)$ a legitimate density function. Plot $f(x)$.
- (b) Find the probability that X is between -2.5 and 2.5
- (c) Find $\mathbb{E}(X)$
- (d) Find σ_X^2
- (e) Compute the CDF of X .
2. In the past few years, the average daily temperature in Socorro on October 10 was $58.5^\circ F$ with standard deviation $9.5^\circ F$. Assume Normal distribution.
- (a) What are the chances that on an October 10, the average daily temperature will be above $40^\circ F$?
- (b) Find the interval $[a, b]$ symmetric around the mean, such that $P(a \leq X \leq b) = 0.98$
3. The number of computer crashes at a Speare lab is believed to have Poisson distribution with the intensity of 0.2 crashes per hour.
- (a) Find the distribution of times between consecutive crashes.
- (b) Find the probability that the lab will go 7 hours without a crash.
- (c) Find the probability that the first crash will occur between 2 and 8 hours.
- (d) Describe the distribution of the time when 3rd crash occurs.
4. The National Highway Traffic Safety Administration claims that 15% of drivers do not use their seat belts. If 200 drivers are randomly checked, what is the probability that at least 25 of them are not wearing seat belts? Find the Normal approximation.
5. The following is a table of joint distribution of X and Y

	Y	0	1	
$X = 1$		0.25	0.35	
$X = 2$		0.1	0.3	

- (a) Are X, Y independent? Explain.
- (b) Fill in the marginal distributions of X, Y
- (c) Find $\mathbb{E}(X)$ and $\mathbb{E}(Y)$
- (d) Find the covariance $\sigma_{X,Y}$
- (e) Let $W = 2X + 3Y$. Find $\mathbb{E}(W)$ and σ_W^2 .

6. An electrical circuit consists of 3 resistors connected in series, so that the total resistance is equal to $Y = R_1 + R_2 + R_3$. Each resistance R_i is random with the mean 900Ω and standard deviation 50Ω .
- What is the highest possible probability that R_1 is outside the $[750 \Omega, 1050 \Omega]$ interval?
 - Find the expected value of Y
 - Find standard deviation of Y (assume that resistors are independent of each other).
 - Suppose that the resistors are no longer independent, but for each i and j , $Cov(R_i, R_j) = 1000 \Omega^2$
 - Find standard deviation of Y now.
 - Find correlation between R_1 and R_2
7. Suppose that X and Y have joint density $f(x, y) = ky$, for $0 \leq y \leq x \leq 2$
- find the constant k that makes f a legitimate density function.
 - find $P(X + Y < 2)$
 - find the conditional density of X given that Y equals 0.5
 - Find the marginal density of X . Are X and Y independent?
 - Write down (do not evaluate) an integral expression for $\mathbb{E}[X \sin(Y)]$.
8. The life length of fuses is Exponential with mean = 300 hours. Assume that fuses behave independently.
- Find the joint density for lifetimes of two such fuses.
 - Find $P(\text{Total life of 2 such fuses} \leq 500 \text{ hours})$.
9. For X, Y given in the scatterplot, visually estimate σ_{X+Y}^2 and σ_{X-Y}^2 . [Hint: first, guess the correlation $\rho_{X,Y}$]

