## Math 382 <br> Practice Exam 1

1. Consider a well-shuffled deck of cards consisting of 52 distinct cards, 4 of which are aces. Draw one card from the deck and put it aside. Then draw another card. Denote $A_{1}=$ "First card is an ace", $A_{2}=$ "Second card is an ace"
(a) Find $P\left(A_{1}\right)$
(b) Find $P\left(A_{2} \mid A_{1}\right)$
(c) Find $P\left(A_{1} \mid A_{2}\right)$
(d) Are events $A_{1}, A_{2}$ independent? Explain.
(e) What is the probability that you draw your first ace on your third try?
2. A computer manufacturer uses chips from three sources, equal amount from each source. Chips from sources A, B and C are defective with probabilities $0.001,0.005$ and 0.01 , respectively.
(a) What proportion of all chips used by manufacturer will be defective?
(b) If a chip is found to be defective, find the probability that the source was C.
3. Multiple choice: circle the appropriate answer and explain.
(a) If the events A and B are mutually exclusive then $\mathrm{A}, \mathrm{B}$ are independent. Is this true?

Always Never Sometimes
(b) If the sample space consists of $n$ outcomes, then each outcome has the probability $1 / n$.

Always Never Sometimes
(c) The following is the graph of CDF of a random variable Y. Then $P(Y=3)$ equals:
$0.5 \quad 0 \quad 0.25 \quad 0.55$

(d) A box contains 6 red balls and 13 white balls. Four balls are drawn at random. The probability that we will get exactly 2 red balls and 2 white ones is

| 0.000258 | 0.0387 | 0.302 | $4 / 19$ |
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4. The average number of accidents at a busy intersection is 41.6 per year.
(a) Find the expected number of accidents in a week.
(b) Find the probability that there will be no accidents in a week
(c) Find the probability that there will be 2 or more accidents in a month
5. An insurance company is selling a "one-sum" policy on your car. You pay a premium of $\$ 500$ in the beginning of the year. In the event your car gets seriously damaged, you will obtain a payment of $\$ 9000$. The probability that a car will get seriously damaged during a year is estimated as $4 \%$.
(a) Find the distribution of the random variable $\mathrm{X}=$ amount of money you receive from the company in a year [make a table containing $x$ and $f(x)$ ].
(b) Find the expected profit of the insurance company from the policy.
(c) If the insurance company sells 500 such policies, what is its expected profit?
(d) Find the variance of the profit.
6. Shaquille O'Neal makes $50 \%$ of his free throws. He attempts 10 in a game.
(a) Find the probability he will make 8 or more.
(b) Find the standard deviation of the number of free throws he makes.
(c) What assumption should one make to be able to evaluate a), b) ?
(d) Find the probability that he needs 5 free throws to make his 2nd.
7. (a) Assuming that components act independently, find the probability that the following system works: (each component has reliability of 0.7)

(b) Suppose the system has $n$ components, all connected in parallel. How large should $n$ be to make the reliability of the entire system higher than $99.9 \%$ ? (each component still has the reliability of 0.7 ) Show your reasoning.
Exam policy: one page ( $8.5 \times 11$ inches) with formulas, calculator
