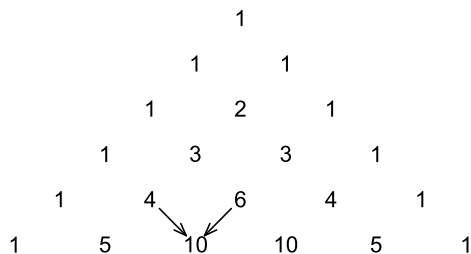


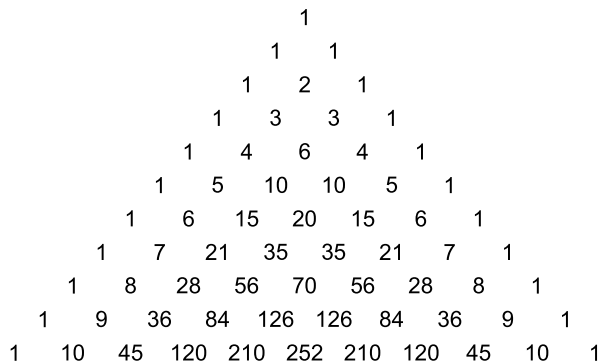
### Pascal's triangle and binomial coefficients

Long before Pascal, this triangle has been described by several Oriental scholars. It was used in the budding discipline of probability theory by the French mathematician Blaise Pascal (1623-1662). The construction begins by writing 1's along the sides of a triangle and then filling it up row by row so that each number is a sum of the two numbers immediately above it.



A step in construction

The number in each cell represents the number of downward routes from the vertex to that point (can you explain why?). It is also a number of ways to choose  $r$  objects out of  $n$  (can you explain why?), that is,  $\binom{n}{r}$ .



The first 10 rows

The combinations numbers are also called *binomial coefficients* and are seen in Calculus. Namely, they are the terms in the expansion

$$(a + b)^n = \sum_{r=0}^n \binom{n}{r} a^r b^{n-r}$$

Note that, if you let  $a = b = 1/2$ , then on the right-hand side of the sum you will get the probabilities

$$P(a \text{ is chosen } r \text{ times and } b \text{ is chosen } n - r \text{ times}) = \frac{\binom{n}{r}}{2^n}$$

and on the left-hand side you will have 1 (the total of all probabilities).