Math 382. Final Exam practice key.

1. (a) 2-sample t-test

(b) $H_0 : \mu_1 = \mu_2, \quad H_A : \mu_1 \neq \mu_2$

(c) $s_p = \sqrt{\dfrac{10.3^2 * 9 + 12.5^2 * 9}{10 + 10 - 2}} = 11.45$, $t = \dfrac{85.2 - 76.0}{11.45\sqrt{1/10 + 1/10}} = 1.797$, using $df = 18$,

p-value is between $2 * 0.05$ and $2 * 0.025$, that is, between 0.1 and 0.05.

(d) Since `p-value` $> \alpha = 0.05$, Accept H_0. We did not find a significant difference between fresh and stored product potency.

2. (a) 1-sample t-test

(b) $H_0 : \mu = 16, \quad H_A : \mu \neq 16$

(c) $t = \dfrac{16.4 - 16}{3.6/\sqrt{25}} = 0.556$, using $df = 24$, p-value is the area under t-curve below 0.556,

clearly it is more than 0.5.

(d) Since `p-value` $> \alpha$, Accept H_0. We did not find that the average time was different from 16.

3. (a) matched pairs test

(b) $H_0 : \mu_D = 0$, $H_A : \mu_D \neq 0$ where μ_D is the average difference between garage I and garage II.

(c) Mean difference $\overline{X} = 240$, $S = 311$. $t = 2.31$, $df = 8$, p-value $\approx .05$, but slightly less. Reject H_0 at level $\alpha = 0.05$, but just barely.

(d) Conclusion: the garage I has, on average, different estimates. Can the t-test be safely used? Maybe not, since the distribution of differences is right-skewed (outliers at 520 and 900).

4. (a) z-test for proportion

(b) $H_0 : p = 0.1$, $H_A : p \neq 0.1$

(c) $\hat{p} = 48/345 = 0.139$, $z = \dfrac{0.139 - 0.1}{\sqrt{0.1 * 0.9/345}} = 2.41$, p-value $= 2 * P(Z > 2.41) = 0.016$

(d) p-value $< \alpha = 0.05$, Reject H_0. Significant evidence that not 10% WonderWidgets are returned (likely more).

5. $t_{\alpha/2} = 1.729$, C.I. is $20.1 \pm 1.729 * 5.3/\sqrt{20} = [18.1, 22.1]$
Looking at the above interval, does it seem credible that $\mu = 18.6$? – Yes, 18.6 belongs to the C.I. Accept H_0 at the level $\alpha = 1 - 0.90 = 0.10$.

6. (a) $z_{\alpha/2} = 1.645$, $n = (1.645 * 3.5/0.3)^2 \approx 368$

(b) $z_{\alpha/2} = 1.96$, margin of error $= .05/2$,
$n = (1.96/0.025)^2 * 0.139 * (1 - 0.139) \approx 736$

7. $P(X \geq 75) = P(\hat{p} \geq 75/400) \approx P\left(Z > \dfrac{.1875 - 0.2}{\sqrt{0.2(1 - 0.2)/400}}\right) \approx P(Z > -0.63) = 0.7357$

8. (a) 900

(b) 12.5

(c) $\approx P(Z > 0.8) = 0.2119$

9. (a) 2000

(b) 200 (you can either use the Gamma formulas, or properties of the Sums of indep. exponentials)

(c) use normal approx., 0.0668

(d) $T = 2000 + 200(-1.645) = 1671$

10. (a) yes

(b) no (success probability not constant)

(c) $\mathbb{E}(T) = 11$, $\sigma^2_{3X+Y} = 3^2\sigma^2_X + \sigma^2_Y = 16.9$, $\sigma_T \approx 4.1$.

11. (a) $K = 1$

(b) $f_X(x) = x/2, 0 < x < 2$, $f_Y(y) = 2(1-y), 0 < y < 1$

(c) $\frac{1}{2(1-y)}, 2y < x < 2$, for the numerical value sub. $y = 0.8$.

(d) 0.5

(e) 8/9

12. (a) 7/15

(b) 14/15

(c) 7/45

13. (a) use $\mu = 0.25 * 7 = 1.75$. $P(N \geq 3) = 1 - e^{-1.75}(1 + 1.75 + 1.75^2/2) \approx 0.256$

(b) if $X \sim \texttt{Exponential}(\beta = 1/0.25 = 4)$, then use $F(2) - F(1) = (1 - e^{-2/4}) - (1 - e^{-1/4}) \approx 0.172$

14. (a) 0.24

(b) 1/4