Final Exam practice. Math 382

I. Statistics

I.a. Hypothesis tests

For each situation below,

- Set up the null and alternative hypotheses.
- State the assumptions.
- Compute test statistic and p-value.
- Make your conclusion in the words of problem (use $\alpha = 0.05$ for all problems).

Names of the Tests

One-sample t-test   Matched Pairs t-test   2-sample t-test (independent samples)   z-test for proportion

1. Company officials are concerned about the length of time a particular drug retains its potency. A random sample of 10 bottles of the product is drawn from current products and analyzed. The average potency of this sample was 85.2 with standard deviation of 10.3. A second sample of 10 bottles is obtained from the current products, stored for one year, and then analyzed. The average potency of this sample was 76.0 with standard deviation of 12.5. We want to test for a difference in the average potency (current and stored).

2. The administrator of a nursing home would like to do a study of staff time spent performing non-emergency type chores. In particular, she would like to test if the average time spent on non-emergency chores is different from 16 hours per day. A random sample of 25 days is taken, yielding the mean of 16.4 hours and standard deviation of 3.6 hours.

3. Insurance adjusters are concerned with the high estimates they are receiving from garage I for auto repairs compared with garage II. To verify their suspicions, each of 9 cars recently involved in an accident was taken to both garages for separate estimates of repair costs. We want to test if the average estimate is higher at garage I than at garage II. The data are given below

Car No.	1	2	3	4	5	6	7	8	9
Garage I	560	40	670	3100	640	1450	3230	520	1930
Garage II	620	50	510	2840	510	1140	2710	570	1030

Hint: calculate the differences between Garage I and Garage II estimates for each car. Then test if the mean difference equals 0.

4. A large online retailer is analyzing how frequently their product `WonderWidget™` is returned. Out of 345 WonderWidgets bought, 48 were returned. Is there evidence that the proportion of WonderWidgets returned is different from 10%? Also, find a 95% confidence interval.

I.b. Confidence intervals, $\overline{X}$, $\hat{p}$

5. Tobacco companies claim that an average smoker usually smokes 18.6 cigarettes a day. To test this, an independent agency took a sample of 20 smokers, on a given day they smoked average of 20.1 cigarettes, with the standard deviation of 5.3 cigarettes. Build a 90% C.I. for the average number of cigarettes μ smoked in a day.

Using C.I. for testing: Looking at the above interval, does it seem credible that $\mu = 18.6$? Test the hypotheses $H_0 : \mu = 18.6$ vs. $H_A : \mu \neq 18.6$ using the C.I. above. Write your conclusion. At what level α would you test?

6. (a) If we knew that the population st.dev. $\sigma = 3.5$, and we wanted to estimate the mean μ within the margin 0.3 with 90% confidence, what sample size would be sufficient for that?

(b) In Problem 4, if we wanted the C.I. with width 0.05, what is the sample size needed?

7. A large retailer estimates that their product `WonderWidget`™ needs service in 20% cases. Out of 400 WonderWidgets bought, what is the probability that 75 or more need service?

8. An electrical circuit consists of 16 resistors. Each resistance R_i is random with the mean 900Ω and standard deviation 50Ω. Let $\overline{X}$ = average resistance of these.

(a) Find the expected value of $\overline{X}$.

(b) Find standard deviation of $\overline{X}$ (assume that resistors are independent of each other).

(c) Approximate the probability that $\overline{X} > 910\Omega$

II. Comprehensive part

9. The lifetime of a workstation (in days) has Gamma distribution with $\alpha = 100$ and $\beta = 20$.

(a) Find the average lifetime of a workstation.

(b) Find the standard deviation of lifetime.

(c) Approximate the probability that the workstation will last for longer than 2300 days.

(d) Find the warranty time T such that only 5% of all workstations fail before T.

10. A test consists of 10 multiple-choice questions, with 5 variants of an answer each, and 10 true/false questions. A person is guessing answers randomly. Let X be the number of correct answers to the first 10 questions and Y the number of correct answers to the last 10 questions.

(a) Are X, Y Binomial?
(b) Let $N = X + Y$ be total number of correct answers. Is it Binomial?
(c) Let the student receive total of $T = 3X + Y$ points for the test. Find $\mathbb{E}(T)$ and σ_T.

11. The random variables X, Y have the following joint density function:

$$f(x,y) = \begin{cases} K, & 0 \le x \le 2, \quad 0 \le y \le 1, \quad \texttt{and} \quad 2y \le x \\ 0 & \texttt{elsewhere} \end{cases}$$

Sketch the region where f is positive and answer the following questions:

(a) Find the constant K

(b) Find marginal densities of X and Y.

(c) Find the conditional density of X, given $Y = 0.8$

(d) Find $P(X \le 1.5, Y \le 0.5)$

(e) Find $P(Y \leq 0.5 \,|\, X \leq 1.5)$

12. Out of 10 thermistors available, 2 are defective. We choose 3 thermistors randomly. Find the probabilities that

(a) none of the chosen are defective

(b) the number of defectives does not exceed 1.

(c) the first defective thermistor will be selected on the 3rd try.

13. Let the number of accidents at a busy intersection follow a Poisson process, with the mean 0.25 accidents per day. Assume that the rate is constant throughout the day.

(a) Find the probability that there will be at least 3 accidents in a week.

(b) Find the probability that the first accident will occur on the second day.

14. The probability that an inoculated person will get flu is 20%. On the other hand, a person not inoculated will get it with probability 60%. It's known that 90% of the population get inoculated.

(a) Find the percentage of population who will get the flu.

(b) Given that a person got the flu, what is the probability that he/she was not inoculated?