Math 382 **Practice Exam 1.**

1. Consider a well-shuffled deck of cards consisting of 52 distinct cards, 4 of which are aces.
Draw one card from the deck and put it aside. Then draw another card.
Denote A1 = "First card is an ace" A2 = "Second card is an ace"

i. Find P(A1) $4/52$

ii. Find P(A2 | A1) $3/51$

iii. Find P(A1 | A2) $3/51$. You could also use Bayes' formula:

$$P(A_1|A_2) = \frac{P(A_1 A_2)}{P(A_2)} = \frac{P(A_2|A_1)\times P(A_1)}{P(A_2|A_1)\times P(A_1) + P(A_2|\overline{A_1})\times P(A_1)}$$

iv. Are events A1, A2 independent? Explain.

no since e.g. $\underset{3/51}{P(A_2|A_1)} \neq \underset{4/52}{P(A_2)}$

v. What is the probability that you draw your first ace on your third try?

$$= P(\overline{A_1}\,\overline{A_2}\,A_3) = \frac{48}{52}\times\frac{47}{51}\times\frac{4}{50}$$

<u>not</u> geometric (no independence)

2. A computer manufacturer uses chips from three sources, equal amount from each source.
Chips from sources A, B and C are defective with probabilities 0.001, 0.005 and 0.01, respectively.

a) What proportion of all chips used by manufacturer will be defective?

$$P(D) = P(D|A)\times P(A) + P(D|B)\times P(B) + P(D|C)\times P(C) =$$
$$= .0053$$ (Bottom part of Bayes' formula)

b) If a chip is found to be defective, find the probability that the source was C.

$$P(C|D) = \frac{P(D|C)\times P(C)}{P(D)} = \frac{.01\times 1/3}{.0053} = 0.629$$

3. Multiple choice: circle the appropriate answer.

a) If the A and B are mutually exclusive then A, B are independent. Is this true?

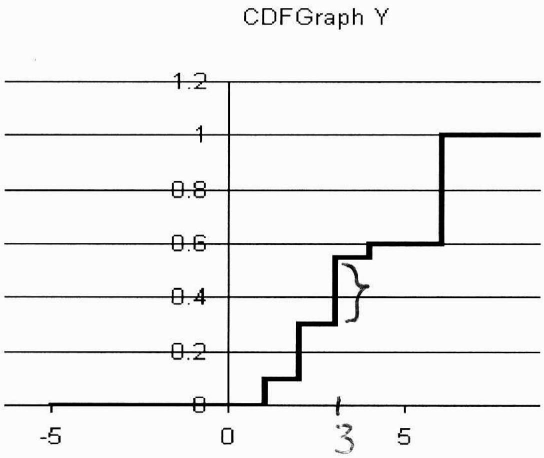
Always Never Sometimes, only if $P(A) = P(B) = 0$

b) If the sample space consists of *n* outcomes, then each outcome has the probability 1/n.

Always Never Sometimes

c) The following is a graph for CDF of Y. Then P(Y = 3) equals:

0.5 0 0.25 0.55



$F(y)$

$P(Y=3) = F(3) - F(2) =$

$= 0.55 - 0.3 = 0.25$

d) A box contains 6 red balls and 13 white balls. Four balls are drawn at random.
The probability that we will get exactly 2 red balls and 2 white ones is

0.000258 0.0387 0.302 4/19

$$\frac{\binom{6}{2}\binom{13}{2}}{\binom{19}{4}} = \frac{\frac{6\times5}{2\times1}\times\frac{13\times1}{2\times1}}{\frac{19\times18\times17\times}{4\times3\times2\times}}$$

e) $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - \frac{1}{k^2} = 0.75$

(Tchebyshelf), use $k = 2$ interval is $52 \pm 2\times4$, is 44 to 60 hours

4. The average number of accidents at a busy intersection is 41.6 per year. (Poisson)

a) Find the expected number of accidents in a week.

$$\frac{41.6}{52} = 0.8$$

b) Find the probability that there will be no accidents in a week

$$P(Y=0) = \frac{0.8^0}{0!} e^{-0.8} = 0.449$$

c) Find the probability that there will be 2 or more accidents in a week

$$P(Y \ge 2) = 1 - [p(0) + p(1)] = 1 - F(1) = 1 - 0.809 =$$

$$= 0.191$$

5. An insurance company is selling a "one-sum" policy on your car. You pay a premium of $500 in the beginning of the year. In the event your car gets seriously damaged, you will obtain a payment of $9000. The probability that a car will get seriously damaged is estimated as 4%.

a) Find the distribution of the random variable X = amount of money you receive from the company in a year

X	p(x)
0	.96
9,000	.04

$E(X) = 360$

b) Find the expected profit of the insurance company from the policy.

$-E(X) + 500 = -9{,}000 \times (.04) + 500 = \$140$

c) If the insurance company sells 500 such policies, what is its expected profit?

profit $= 140 \times 500 = \$70{,}000$

d) Find the variance of the profit. ← do part (b)

$\text{Profit} = \cancel{1} \cdot 500 - X, \quad Var(\text{Profit}) = (-1)^2 Var(X)$

$V(X) = \sum x^2 p(x) - \mu^2 = 9{,}000^2 \times .04 - 360^2 = 3.1 \times 10^6$

6. Shaquille O'Neal makes 50% of his free throws. He attempts 10 in a game.

a) Find the probability he will make 8 or more.

Binomial $n = 10, \ p = 0.5$

$P(X \geq 8) = 1 - F(7) = 1 - .945 = .055$

b) Find the standard deviation of the number of free throws he makes.

$\sigma = \sqrt{np(1-p)} = 1.58$

c) What assumption should one make to be able to evaluate a), b) ?

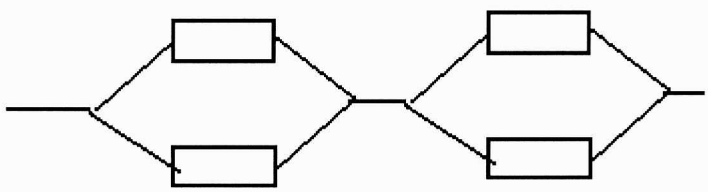
independent attempts

d) Find the probability that he needs 5 free throws to make his 2nd.

Neg. Bin.

$P(Y=5) = \binom{4}{1} 0.5^2 (1-0.5)^3 = 0.125$

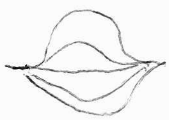
7. a) Assuming that components act independently, find the probability that the following system works: (each component has reliability of 0.7)



$(1-0.3^2)^2 = 0.8281$

$\uparrow$ $1-0.3^2$ $\qquad$ $\uparrow$ $1-0.3^2$

b) Suppose the system has *n* components, all connected in parallel. How large should *n* be to make the reliability of the entire system higher than 99.9%? (each component has reliability of 0.7)
Show your reasoning.



P(At least one) = 1 − P(none work) =

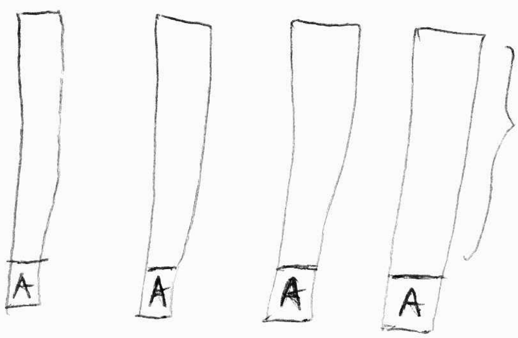
$= 1 - 0.3^n > .999$

$0.3^n \le .001 \qquad n \ln(0.3) \ge \ln(.001)$

$n \ge 5.7$ or 6.

Extra credit.

Refer to problem ~~2~~ 1. Suppose that the entire deck is distributed equally among four players. Find the probability that each player gets an ace.



12 other cards

ace $\qquad$ other cards

$$\frac{4\times\binom{48}{12}\times 3\times\binom{36}{12}\times 2\times\binom{24}{12}\times 1\binom{12}{12}}{\binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13}}$$

all possible choices

≈ 0.1055