Practice Questions for Exam 4 **Math 231**

1. Find *curl* $\vec{F}$ and *div* $\vec{F}$ if $\vec{F}(x,y,z)=e^{-x}\sin y\,\mathbf{i}+e^{-y}\sin z\,\mathbf{j}+e^{-z}\sin x\,\mathbf{k}$

2. Evaluate the line integral $\int_C x^3 z\,ds$ where C is the curve $\vec{r}(t)=(2\sin t)\mathbf{i}+t\mathbf{j}+(2\cos t)\mathbf{k}$ for $0\le t\le \pi/2$.

3. Evaluate $\int_S (x^2z+y^2z)dS$ where S is part of the plane $z=4+x+y$ that lies inside the rectangle $0\le x\le 1,\ 0\le y\le 1$.

4. Use Green's Theorem to evaluate $\int x^2y\,dx-xy^2\,dy$ where C is the circle $x^2+y^2=4$ with counterclockwise orientation.

5. Use Stokes' Theorem to evaluate $\iint_S (curl\,\vec{F})\cdot\vec{n}\,dS$ where $\vec{F}(x,y,z)=\left\langle x^2yz, yz^2, z^3e^{xy}\right\rangle$, S is part of the sphere $x^2+y^2+z^2=5$ that lies above the plane $z=1$ and S is oriented upward.

6. Evaluate the line integral $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F}=(4x^3y^2-2xy^3)\mathbf{i}+(2x^4y-3x^2y^2+4y^3)\mathbf{j}$ and C: $\vec{r}(t)=(t+\sin\pi t)\mathbf{i}+(2t+\cos\pi t)\mathbf{j}$, $0\le t\le 1$.

7. Evaluate the line integral $\int_C (xy+\ln x)dy$ where C is the arc of the parabola $y=x^2$ from (1, 1) to (3, 9).

8. Use Green's Theorem to evaluate the line integral $\int_C \left(y+e^{\sqrt{x}}\right)dx+\left(2x+\cos y^2\right)dy$ along the positively oriented curve C where C is the boundary of the region enclosed by the parabolas $y=x^2$ and $x=y^2$.

9. Show that the vector field $\vec{F}(x,y,z)=\left(2xz+y^2\right)\mathbf{i}+2xy\,\mathbf{j}+\left(x^2+3z^2\right)\mathbf{k}$ is conservative and evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C: $x=t^2,\ y=t+1,\ z=2t-1$ for $0\le t\le 1$.

10. Evaluate the surface integral $\int_S (x^2 + y^2) dS$ where S is the surface $z = xy$ inside $x^2 + y^2 = 4$ for $x \geq 0,\ y \geq 0$.

11. Evaluate $\iint_S curl\,\vec{F} \cdot \vec{n}\, dS$ where $\vec{F}(x,y,z) = x\,\mathbf{i} + y^2\,\mathbf{j} + xyz\,\mathbf{k}$ and S is part of the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$. Use the upward unit normal vector.

12. Find the surface area of the cap cut from the paraboloid $y^2 + z^2 = 3x$ by the plane $x = 1$.

For additional problems, check out the review problems for Chapter 14. Note the questions above are simply a sample of questions possible for the exam; it is possible that other types of questions may appear on your exam.