1. Describe/sketch the region whose area is given by the integral $\int_0^{\frac{\pi}{2}}\int_0^{2\sin\theta} r\,dr\,d\theta$.

   Solution: Right side of a circle centered at $(0,1)$ with radius 1.

2. Write $\iint_R f(x,y)dA$ as an iterated integral where R is the region shown below and f is an arbitrary continuous function on R.

   Solution: $\int_0^{\pi}\int_2^{4} f\left(r\cos\theta, r\sin\theta\right) r\,dr\,d\theta$

3. Rewrite the integral $\int_{-1}^{1}\int_{x^2}^{1}\int_0^{1-y} f(x,y,z)\,dz\,dy\,dx$ as an iterated integral in order $dxdydz$.

   Solution: $\int_{-1}^{1}\int_{x^2}^{1}\int_0^{1-y} f(x,y,z)\,dz\,dy\,dx = \int_0^1\int_0^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} f\left(x,y,z\right)dx\,dy\,dz$

4. Find the mass of the plane lamina bounded by $x = 2y^2$ and $y^2 = x-4$ with density $\delta(x,y) = y^2$.

   Solution: $\int_{-2}^{2}\int_{2y^2}^{y^2+4} y^2\,dx\,dy = \frac{128}{15}$

5. Set up the iterated triple integral, complete with correct limits, for the volume of the solid bounded by $y = 4 - x^2 - z^2$, $x = 0$, $y = 0$, $z = 0$, and $x + z = 2$. Do not evaluate the integral.

   Solution: $V = \int_0^2\int_0^{2-x}\int_0^{4-x^2-z^2} dy\,dz\,dx$

6. For the following double integral, sketch the region, reverse the order of integration, then evaluate.

   Solution: $\int_0^3\int_{x^2}^{9} x\cos(y^2)\,dy\,dx = \int_0^9\int_0^{\sqrt{y}} x\cos\left(y^2\right)dx\,dy = \frac{1}{4}\sin\left(81\right)$

7. Set up the iterated triple integral, with correct limits, to find the volume of the ice cream cone bounded above by the sphere $\rho = a$ and below by the cone $\phi = \frac{\pi}{3}$.

   Solution: $V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^{a} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

8. Find the volume of the solid inside the sphere $x^2+y^2+z^2=16$ and outside the cylinder $x^2+y^2=4$.

Solution: $V=2\int_0^{2\pi}\int_2^4\int_0^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta=32\sqrt{3}\pi$

9. Express the triple integral $\iiint_T f(x,y,z)dV$ where T is the solid bounded by the surfaces $x^2+z^2=4,\ y=0,\ y=6$.

a. Set up the integral first integrating with respect to *x*, second with respect to *z* and then with respect to *y*.

Solution: $\int_0^6\int_{-2}^2\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} f(x,y,z)\,dx\,dz\,dy$

b. Set up the integral first integrating with respect to *y*, second with respect to *z* and then with respect to *x*.

Solution: $\int_{-2}^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_0^6 f(x,y,z)\,dy\,dz\,dx$

10. Describe a solid whose volume is equal to $\int_0^{2\pi}\int_0^3\int_{-\sqrt{9-r^2}}^0 r\,dz\,dr\,d\theta$.

Solution: Sold is the bottom half of a sphere centered at the origin with a radius of three.

11. Convert to spherical coordinates and evaluate.

$$\int_{-2}^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} e^{-\left(x^2+y^2+z^2\right)^{3/2}}\,dz\,dy\,dx=\int_0^{2\pi}\int_0^{\pi/2}\int_0^2 e^{-\rho^3}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta=\frac{2\pi}{3}\left(1-e^{-27}\right)$$

12. Set up the integral to find the area of the first quadrant region bounded by the lines $y=x,\ y=2x$ and the hyperbolas $xy=1,\ xy=2$, using the substitute $u=xy,\ v=\frac{y}{x}$.

Solution: $A=\iint_R dA=\int_1^2\int_1^2\frac{1}{2v}\,du\,dv$