

1. Find the equation of the plane passing through the points $(1, 2, 1)$, $(-1, 3, 2)$, and $(0, -1, 5)$.
2. Determine whether the planes $3x + 2y - 3z = 10$ and $-6x - 10y + z = 10$ are parallel, orthogonal or neither. If neither, what is the angle between the two planes?
3. Find an equation of the line of intersection of the planes $Q : 2x - y + 3z - 1 = 0$ and $R : -x + 3y + z - 4 = 0$
4. Consider the cylinder $x = z^2 - 4$ in \mathbb{R}^3 . Identify the coordinate axis to which the cylinder is parallel. Sketch the cylinder.
5. Identify and briefly describe the surface $x^2 + y^2 + z^2 + 2x - 4y - 16 = 0$.
6. Identify and briefly describe the surface $y = 4x^2 + z^2$.
7. Find $\frac{\partial z}{\partial x}$ for $xe^{yz} + ye^{xz} + ze^{xy} = 5$, assuming that $z = f(x, y)$
8. Find the linear approximation to the function, $f(x, y) = \sqrt{x^2 + y^2}$ at the point $P(3, -4)$. Use it to estimate $f(3.06, -3.92)$.
9. Sketch the domain of the function $f(x, y) = \sqrt{y-x} \ln(y+x)$ in the xy -plane.
10. Given the function $f(x, y, z) = e^{xy^2z^3}$
 - a. Find $\frac{\partial f}{\partial x}$
 - b. Find $\frac{\partial^2 f}{\partial x^2}$
 - c. Find $\frac{\partial^2 f}{\partial x \partial y}$
 - d. Find $\frac{\partial^3 f}{\partial z \partial y \partial x}$
11. Find the directional derivative to the surface given by the function $f(x, y) - 7 + 10x\sqrt{y}$ at the point $P(5, 16)$ in the direction of the vector $\vec{v} = \langle -4, 3 \rangle$.
12. Find an equation of the plane tangent to the surface given by the function $f(x, y) = x^3 - x^2y$ at the point $P(2, 1)$.
13. For $f(x, y) = x^2 - y^2$, find a line in the x direction tangent to the surface defined by f at $(1, 2)$.
14. Suppose that the temperature w (in degrees Celsius) at the point (x, y) is given by $w = f(x, y) = 5 + 0.002x^2 + 0.003y^2$. In what direction should the grasshopper hop from the point $(10, 20)$ to get warmer as quickly as possible? What is the rate of change of the temperature in this direction?
15. Find the directional derivative to the function $f(x, y, z) = 2z\sqrt{xy}$ at the point $(2, 2, 3)$ in the direction of the vector $\mathbf{v} = \langle 1, 1, 1 \rangle$.

16. Use the method of Lagrange multipliers to find the dimensions of a rectangular box (with a lid) with largest volume, if the total surface area is 96 cm^2 .

17. Find $\frac{\partial z}{\partial t}$ for $z = \sin(x^2y)$, $x = \frac{s}{t}$, and $y = t^2e^{st}$.

18. Find the limit or state that it does not exist:

$$\lim_{(x,y) \rightarrow (0,0)} e^{\sin(x+y-\pi/2)}$$

19. Show the following limit does not exist:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{4xy}{3x^2 + y^2}$$

20. Find ∇f for $f(x, y, z) = xy + xz + yz + 4$.

21. Find the direction in which the function $f(x, y, z) = xe^z - ye^x$ decreases most rapidly from the point $P(0, 2, 0)$.

22. For the function $f(x, y) = x^2 - y$, make a sketch of several level curves. Label at least two level curves with their z -values.

23. A rectangular box has a square base. Find the rate at which its volume is changing if its base edge is increasing at 2 cm/min and its height is decreasing at 3 cm/min at the instant when each dimension is 1 meter .

24. Use the method of Lagrange Multipliers to find the minimum value of the function $f(x, y) = x^2 + y + 2z$ subject to the constrain $x^2 + 2y^2 + z^2 = 1$.

25. Locate and classify the critical points of the function $f(x, y) = 3xy - x^2y - xy^2$.

26. According to postal regulations, the largest cylindrical container that can be sent has girth $2\pi r$ plus length l of 108 inches . What is the largest cylindrical container that can be mailed?

For additional problems, check out the review problems for Chapter 12. Note the questions above are simply a sample of possible questions possible for the exam; it is possible that other types of questions may appear on your exam.