Exam 4 **Name**________________________________

Math 231-01 **Calculus III** **August 2, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 65 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.
- $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos$ and $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin$
- $x = \rho \sin . \cos$, $y = \rho \sin \sin$, $z = \rho \cos$, and $dV = \rho^2 \sin d\rho\, d\varphi\, d\theta$
- **Green's Theorem** Let C be a closed bounded curve that bounds a region R in the plane and let $\mathbf{F}(x, y)) = f\,\mathbf{i} + g\,\mathbf{j}$ be a vector field then

  1. Circulation: $\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C f\,dx + g\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA.$
  2. Outward Flux Integral: $\oint_C \mathbf{F} \cdot \mathbf{n}\,ds = \oint_C f\,dy - g\,dx = \iint_R \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dA$

- **Stokes' Theorem** Let S be an oriented surface in $\mathbb{R}^3$ with a piecewise-smooth closed boundary C whose orientation is consistent with that of S. Assume that $\mathrm{F} = \langle f, g, h \rangle$ is a vector field whose components have continuous first partial derivatives on S. Then

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

  when n is the unit vector normal to S determined by the orientation of S.

Question	Points	Score
1	8	
2	10	
3	10	
4	10	
5	10	
6	10	
7	12	
8	10	
9	10	
10	10	
Total:	100	

1. (8 points) Complete test corrections.

2. (10 points) Evaluate the line integral $\int_C (y - z)\,ds$ where C is the helix $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, t\rangle$ for $0 \le t \le 2\pi$.

3. (10 points) Compute the divergence and curl of the vector field $\mathbf{F} = \langle 2xy + z^4, x^2, 4xz^3\rangle$. State whether the field is source free or irrotational.

4. (10 points) Use a surface integral to find the area of the surface of a frustum of the cone $z^2 = x^2 + y^2$ for $2 \leq z \leq 4$ (excluding the bases).

5. (10 points) Evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ using Stokes' Theorem where $\mathbf{F} = \langle x^2 - y^2, x, 2yz \rangle$; C is the boundary of the plane $z = 6 - x - y$ in the first octant. Assume that C has counterclockwise orientation. Set up but do not evaluate.

6. (10 points) Evaluate $\oint_C (3y - e^{\sin x})\,dx + (7x + \sqrt{y^4+1})\,dy$, where C is the circle $x^2 + y^2 = 9$.

7. Given $\mathbf{F} = \langle y^2z + 2xz^2, 2xyz, xy^2 + 2x^2z \rangle$ and C is $\mathbf{r}(t) \left\langle \sqrt{t}, t+1, t^2 \right\rangle$ for $0 \le t \le 1$.

   a. (6 points) Find a function φ such that $\mathbf{F} = \nabla\varphi$

   b. (6 points) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

8. (10 points) Use Stokes' Theorem to evaluate the surface integral $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS$ where $\mathbf{F} = \langle x^2 - y^2, y^2, xz \rangle$; S is the hemisphere $x^2 + y^2 + z^2 = 4$ for $y \geq 0$. The normal vector is outward, points to positive y-axis.

9. (10 points) Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$ for $\mathbf{F} = \langle x, y, z^2 \rangle$; S is the sphere with radius 1 and center at the origin. Set up, but do not evaluate.

10. (10 points) Evaluate the line integral $\int_C \mathbf{F} \cdot \mathbf{T}\, ds$ for $\mathbf{F} = \langle xy^2, -x^2 \rangle$; C is $\mathbf{r}(t) = \langle t^3, t^2 \rangle$ for $0 \leq t \leq 1$.