Exam 3

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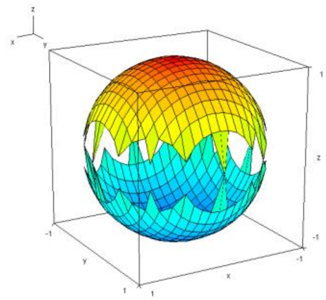
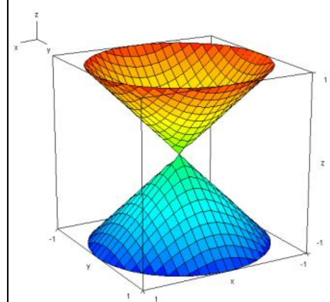
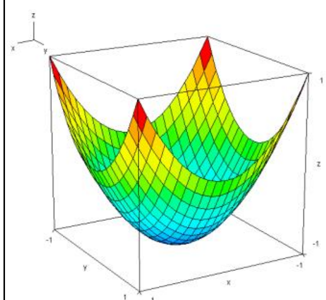
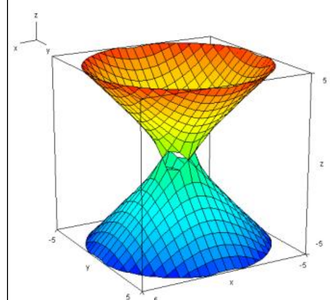
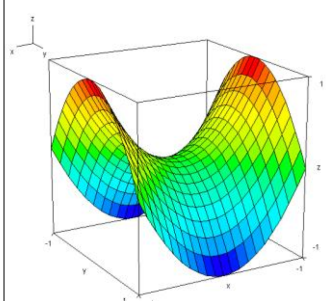
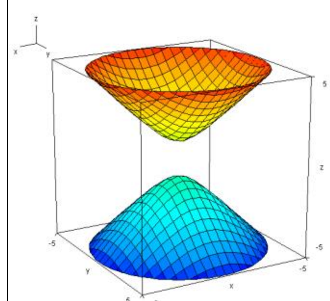
Math 231

Calculus III

Name Solution

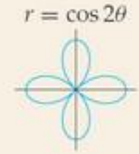
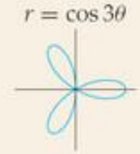
April 9, 2018

Equation	Surface	Equation	Surface
Ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$		Cone $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=\dfrac{z^2}{c^2}$	
Elliptic Paraboloid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=\dfrac{z}{c}$		Hyperboloid 1 Sheet $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-\dfrac{z^2}{c^2}=1$	
Hyperbolic Paraboloid $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=\dfrac{z}{c}$		Hyperboloid 2 Sheets $-\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$	

$x=\rho\sin\phi\cos\theta,\ z=\rho\cos\phi$

$y=\rho\sin\phi\sin\theta,\ dV=\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

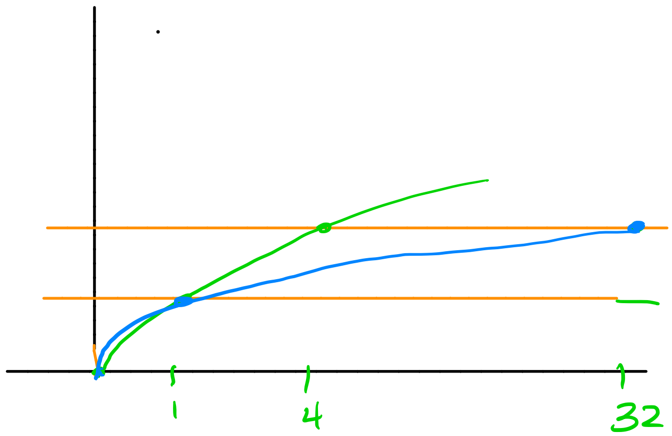
Recall these graphs of polar equations:

Equation	Name	Example
$r = a \pm b\cos\theta$ $r = a \pm b\sin\theta$, where $a,\ b \in \mathbb{R}$, $a \neq 0,\ b \neq 0$	Cardioid when $\left\|\frac{a}{b}\right\| = 1$	$r = 1 + \cos\theta$ 
	Limaçon when $\left\|\frac{a}{b}\right\| \neq 1$ ▶ with inner loop when $\left\|\frac{a}{b}\right\| < 1$	$r = 1 + 2\cos\theta$ 
	▶ dimpled when $1 < \left\|\frac{a}{b}\right\| < 2$	$r = 3 + 2\cos\theta$ 
	▶ convex when $\left\|\frac{a}{b}\right\| > 2$	$r = 5 + 2\cos\theta$ 
$r = a\sin k\theta$ $r = a\cos k\theta$, where $a \in \mathbb{R}$ and $k \in \mathbb{Z}$	Circle when $\|k\| = 1$	$r = \sin\theta$ 
	Rose curve with $2k$ petals when $k \neq 0$ is even	$r = \cos 2\theta$ 
	Rose curve with k petals when $k \neq \pm 1$ is odd	$r = \cos 3\theta$ 
$r^2 = \pm a\sin 2\theta$ $r^2 = \pm a\cos 2\theta$, where $a \in \mathbb{R}$ and $a \neq 0$	Lemniscate	$r^2 = \sin 2\theta$  $r^2 = \cos 2\theta$ 

Show all work! Do not evaluate 1-6.

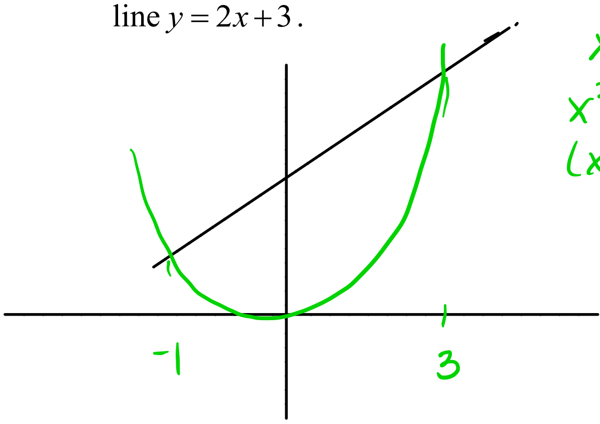
1. (8 points) Sketch the region of integration and reverse the order of integration for

$$\int_1^2 \int_{y^2}^{y^5} e^{x/y^2}\,dxdy$$



$$= \int_1^4 \int_{x^{1/5}}^{x^{1/2}} e^{x/y^2}\,dy\,dx + \int_4^{32} \int_{x^{1/5}}^{2} e^{x/y^2}\,dy\,dx$$

2. (8 points) Use the double integral to find the area of the region bounded by the curve $y = x^2$, and the line $y = 2x + 3$.



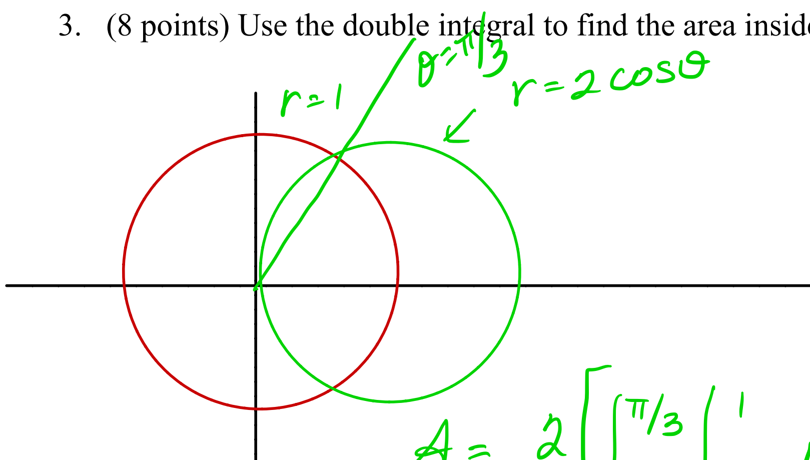
$x^2 = 2x+3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$$A = \int_{-1}^{3} \int_{x^2}^{2x+3} dy\,dx$$

3. (8 points) Use the double integral to find the area inside both circles $r = 2\cos\theta$ and $r = 1$.

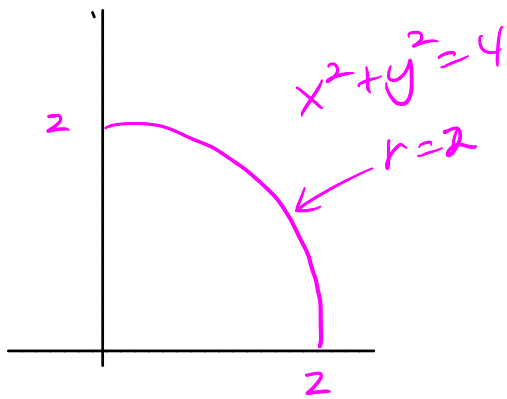


$1 = 2\cos\theta \Rightarrow \cos\theta = \frac{1}{2}$

$\theta = \pi/3$

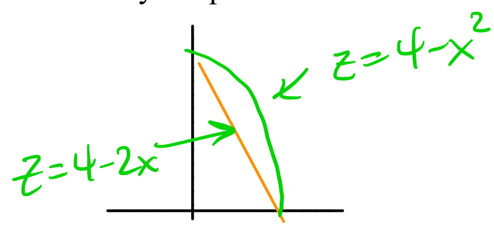
$$A = 2\left[\int_0^{\pi/3}\int_0^1 r\,dr\,d\theta + \int_{\pi/3}^{\pi/2}\int_0^{2\cos\theta} r\,dr\,d\theta\right]$$

4. (8 points) Sketch the region of integration and convert $\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9-x^2-y^2}}\,dx\,dy$ to polar form.



$$\int_0^{\pi/2}\int_0^2 \frac{1}{\sqrt{9-r^2}}\, r\,dr\,d\theta$$

5. (16 points) Find the volume bounded above by $z=4-x^2-y^2$ and below by the plane $z=4-2x$.
   a. Set up the integral in the order $dydzdx$.
   b. Set up the integral in the order $dxdzdy$.



a) $$V=\int_0^2\int_{4-2x}^{4-x^2}\int_{-\sqrt{4-x^2-z}}^{\sqrt{4-x^2-z}} 1\,dy\,dz\,dx$$

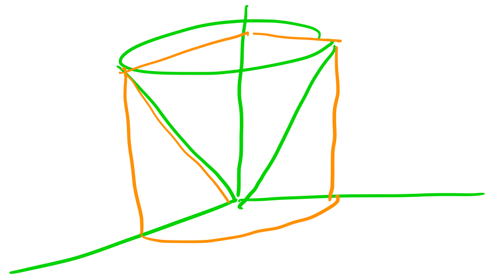
b) $$\int_{-1}^{1}\int_{2-\sqrt{4-4y^2}}^{2+\sqrt{4-4y^2}}\int_{2-\frac{1}{2}z}^{\sqrt{4-y^2-z}} 1\,dx\,dz\,dy$$

Intersect

$4y^2+(z-2)^2=4$

6. (24 points) Write an iterated triple integral for the integral of $f(x,y,z)=6+4y$ over the region in the first octant bounded by the cone $z=\sqrt{x^2+y^2}$, the cylinder $x^2+y^2=1$ and the coordinate planes in:

   a. Rectangular coordinates
   b. Cylindrical coordinates
   c. Spherical coordinates.



a) $$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} (6+4y)\,dz\,dy\,dx$$

b) $$\int_0^{\frac{\pi}{2}} \int_0^1 \int_0^r (6+4r\sin\theta)\,r\,dz\,dr\,d\theta$$

c) $$\int_0^{\pi/2} \int_{\pi/4}^{\pi/2} \int_0^{\csc\phi} (6+4\rho\sin\phi\sin\theta)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

7. (10 points) Evaluate the integral by changing the order of integration in an appropriate way

$$\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2}\,dydxdz$$

$$= \int_0^1\int_0^1\int_0^{\sqrt{y}} 12xze^{zy^2}\,dxdydz$$

$$= \int_0^1\int_0^1 6yze^{zy^2}\,dydz = 3\int_0^1 e^{zy^2}\Big|_0^1\,dz$$

$$= 3\int_0^1 (e^z-1)\,dz = 3(e^z-z)\Big|_0^1 = 3(e^1-1-1)$$

$$= \boxed{3(e-2)}$$

8. (10 points) Evaluate $\iint_R e^{xy}\,dA$, where R is region bounded by the hyperbolas $xy=1$ amd $xy=4$ and the lines $y/x=1$ and $y/x=3$. Use the transformation $u=xy$ and $v=y/x$.

$y = vx \qquad u = x\,vx = x^2v \qquad x^2 = \frac{u}{v} \Rightarrow x = u^{1/2}v^{-1/2}$

$y = v(u^{1/2}v^{-1/2}) = u^{1/2}v^{1/2}$

$$J = \begin{vmatrix} \frac{1}{2}u^{-1/2}v^{-1/2} & -\frac{1}{2}u^{1/2}v^{-3/2} \\ \frac{1}{2}u^{-1/2}v^{1/2} & \frac{1}{2}u^{1/2}v^{-1/2} \end{vmatrix} = \frac{1}{4}\cdot\frac{1}{v} + \frac{1}{4}\,\frac{1}{v} = \frac{1}{2v}$$

$$\int_1^3\int_1^4 e^u\cdot\frac{1}{2}\cdot\frac{1}{v}\,dudv = \frac{1}{2}\int_1^3 \frac{1}{v}(e^4-e)\,dv$$

$$= \frac{1}{2}\ln 3\,(e^4-e).$$