Exam 3

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Math 231

Name______________________________

Calculus III

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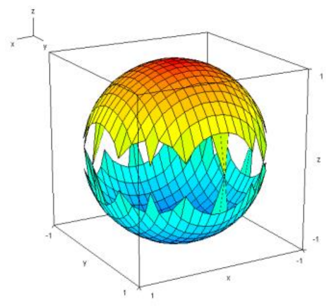
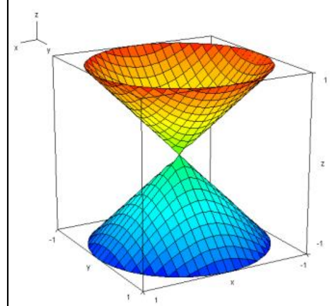
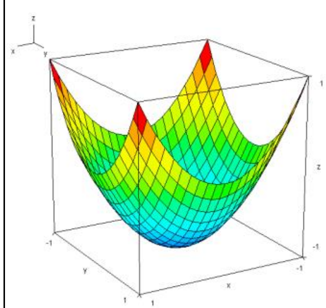
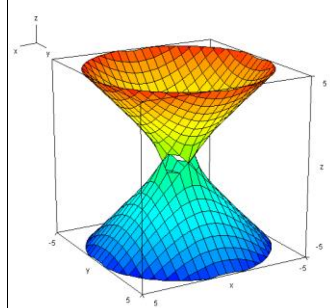
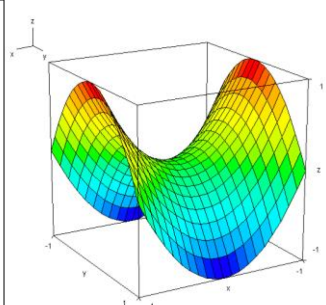
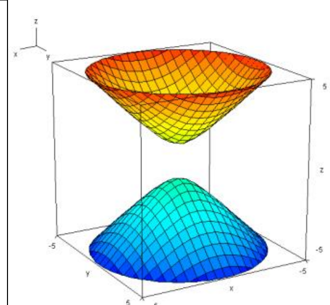
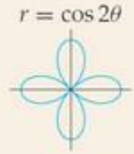
Equation	Surface	Equation	Surface
Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$		Cone $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$	
Elliptic Paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$		Hyperboloid 1 Sheet $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	
Hyperbolic Paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{z}{c}$		Hyperboloid 2 Sheets $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	

$x=\rho\sin\phi\cos\theta,\ z=\rho\cos\phi$

$y=\rho\sin\phi\sin\theta,\ dV=\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

Recall these graphs of polar equations:

Equation	Name	Example
$r = a \pm b\cos\theta$ $r = a \pm b\sin\theta$, where $a,\ b \in \mathbb{R}$, $a \neq 0,\ b \neq 0$	Cardioid when $\left\|\frac{a}{b}\right\| = 1$	$r = 1 + \cos\theta$ 
	Limaçon when $\left\|\frac{a}{b}\right\| \neq 1$ ▶ with inner loop when $\left\|\frac{a}{b}\right\| < 1$	$r = 1 + 2\cos\theta$ 
	▶ dimpled when $1 < \left\|\frac{a}{b}\right\| < 2$	$r = 3 + 2\cos\theta$ 
	▶ convex when $\left\|\frac{a}{b}\right\| > 2$	$r = 5 + 2\cos\theta$ 
$r = a\sin k\theta$ $r = a\cos k\theta$, where $a \in \mathbb{R}$ and $k \in \mathbb{Z}$	Circle when $\|k\| = 1$	$r = \sin\theta$ 
	Rose curve with $2k$ petals when $k \neq 0$ is even	$r = \cos 2\theta$ 
	Rose curve with k petals when $k \neq \pm 1$ is odd	$r = \cos 3\theta$ 
$r^2 = \pm a\sin 2\theta$ $r^2 = \pm a\cos 2\theta$, where $a \in \mathbb{R}$ and $a \neq 0$	Lemniscate	$r^2 = \sin 2\theta$  $r^2 = \cos 2\theta$ 

Show all work! Do not evaluate 1-6.

1. (8 points) Sketch the region of integration and reverse the order of integration for

$$\int_{1}^{2}\int_{y^2}^{y^5} e^{x/y^2}\,dx\,dy$$

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2. (8 points) Use the double integral to find the area of the region bounded by the curve $y = x^2$, and the line $y = 2x + 3$.

3. (8 points) Use the double integral to find the area inside both circles $r = 2\cos\theta$ and $r = 1$.

4. (8 points) Sketch the region of integration and convert $\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9-x^2-y^2}}\,dxdy$ to polar form.

5. (16 points) Find the volume bounded above by $z = 4 - x^2 - y^2$ and below by the plane $z = 4 - 2x$.
    a. Set up the integral in the order $dydzdx$.
    b. Set up the integral in the order $dxdzdy$.

6. (24 points) Write an iterated triple integral for the integral of $f(x, y, z) = 6 + 4y$ over the region in the first octant bounded by the cone $z = \sqrt{x^2 + y^2}$, the cylinder $x^2 + y^2 = 1$ and the coordinate planes in:

   a. Rectangular coordinates
   b. Cylindrical coordinates
   c. Spherical coordinates.

7. (10 points) Evaluate the integral by changing the order of integration in an appropriate way

$$\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2}\,dydxdz$$

8. (10 points) Evaluate $\iint_R e^{xy}dA$, where R is region bounded by the hyperbolas $xy=1$ amd $xy=4$ and the lines $y/x=1$ and $y/x=3$. Use the transformation $u=xy$ and $v=y/x$.