Exam 2 **Name**______________________________

Math 231-02 **Calculus III** **October 10, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 65 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.
- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos\theta$ and $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin\theta$

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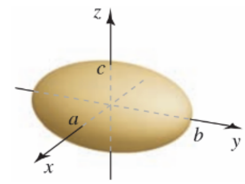
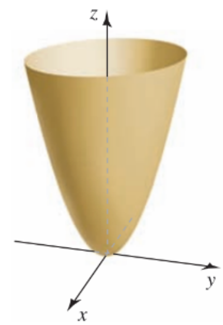
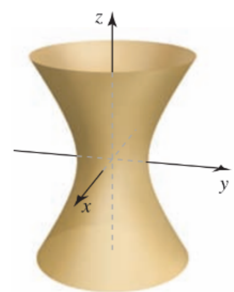
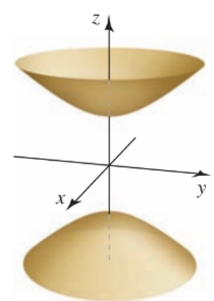
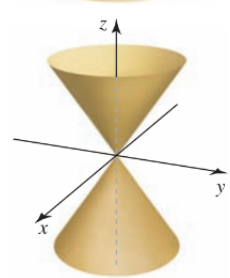
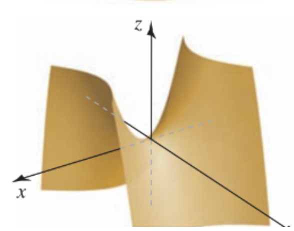
1. (8 points) To be completed once exams are graded and returned. Please correct any problem with points deducted. All corrections should be completed neatly on a separate sheet of paper. Once you have finished your corrections, take your exam and corrections to the Office of Student Learning (OSL), and a tutor will check your answers and sign below. The checked solutions should be given to your instructor.

   Signature: ______________________________
   Print Name: ______________________________
   Date: _______________

Table 12.1

Name	Standard Equation	Features	Graph
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	All traces are ellipses.	
Elliptic paraboloid	$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Traces with $z = z_0 > 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are parabolas.	
Hyperboloid of one sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ are ellipses for all z_0. Traces with $x = x_0$ or $y = y_0$ are hyperbolas.	
Hyperboloid of two sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ with $\|z_0\| > \|c\|$ are ellipses. Traces with $x = x_0$ and $y = y_0$ are hyperbolas.	
Elliptic cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$	Traces with $z = z_0 \neq 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are hyperbolas or intersecting lines.	
Hyperbolic paraboloid	$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$	Traces with $z = z_0 \neq 0$ are hyperbolas. Traces with $x = x_0$ or $y = y_0$ are parabolas.	

Question	Points	Score
1	8	
2	8	
3	6	
4	6	
5	8	
6	8	
7	8	
8	8	
9	8	
10	14	
11	10	
12	8	
Total:	100	

2. (8 points) Find and sketch the domain of $f(x, y) = \sqrt{x^2 - y}$.

3. (6 points) Find the limit $\lim_{(x,y)\to(1,1)} \frac{x - y}{x^2 - y^2}$.

4. (6 points) Show that the limit $\lim_{\substack{(x,y)\to(0,0) \\ y\neq x^2}} \frac{y}{x^2 - y}$ does not exist.

5. (8 points) If $w = xe^{yz}$, compute $\dfrac{\partial^3 w}{\partial y \partial x \partial z}$.

6. (8 points) Find an equation of the tangent plane to surface $x^2 + 2xy - y^2 + z^2 = 7$ at the point $P(1, -1, 3)$.

7. (8 points) Write a Chain Rule formula for $\dfrac{\partial w}{\partial s}$ for $w = g(x, y)$, $x = h(r, s, t)$, and $y = k(r, s, t)$.

8. (8 points) Find the derivative of the function $\cos(xy) + e^{yz} + \ln(xz)$ at the point $P(1, 0, 1/2)$ in the direction $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

9. (8 points) Is $f(x, y) = x^2 - xy + y^2 - 3$ more sensitive to changes in x or to changes in y when it is near the point $(1, 2)$? Use differentials to justify your answer.

10. (14 points) Find the local maxima, local minima, and saddle points of $f(x, y) = 2x^3 + 2y^3 - 9x^2 + 3y^2 - 12y$.

11. (10 points) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x^2y^2z$, assuming $x \geq 0$ and $y \geq 0$, subject to the constraint $2x^2 + y^2 + z^2 = 25$.

12. (8 points) Find the direction in which the function $h(x, y, z) = \ln(x^2+y^2-1)+y+6z$ increase and decrease the most rapidly at $P(1, 1, 0)$. What is the rate of change in these directions?