Exam 1 **Name**______________________________

Math 231-01 **Calculus III** **February 14, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 65 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.
- $\vec{u} \cdot \vec{v} = ||\vec{u}||||\vec{v}|| \cos\theta$ and $||\vec{u} \times \vec{v}|| = ||\vec{u}||||\vec{v}|| \sin\theta$

Question	Points	Score
1	8	
2	12	
3	12	
4	12	
5	6	
6	6	
7	12	
8	8	
9	12	
10	12	
11	0	
Total:	100	

1. (8 points) Complete test corrections.

2. Let $\vec{PQ}$ extend from $P(2, 0, 6)$ and $Q(2, -8, 5)$

   a. (4 points) Find the position vector $\vec{PQ}$.

   b. (4 points) Find the the midpoint M on the line segment PQ. Then find the magnitude of $\vec{PM}$.

   c. (4 points) Find a vector of length 8 with direction opposite of that of $\vec{PQ}$.

3. Let $\vec{u} = \langle 0, 3, -4 \rangle$ and $\vec{v} = \langle -5, 6, 0 \rangle$

   a. (4 points) Compute $\vec{u} \cdot \vec{v}$.

   b. (4 points) Find the angle between $\vec{u}$ and $\vec{v}$.

   c. (4 points) Find $\text{proj}_u v$

4. Consider three points $P(2, -3, 5)$, $Q(3, 1, 0)$ and $R(-1, 0, 7)$

   a. (4 points) Find an equation for the line containing P and Q.

   b. (4 points) Find an equation for the plane containing P, Q, and R.

   c. (4 points) Find the area of the triangle determined by P, Q, and R.

5. (6 points) Find the equation of the sphere with center $(2, -3, 4)$ and tangent to the xz-plane.

6. (6 points) Find the $\lim_{t \to -3} \left\langle \frac{|t+3|}{t+3}, \frac{|t+3|}{t-3} \right\rangle$, if it exists. If it does not exist, explain why.

7. (12 points) For the vector curve $\vec{r}(t) = \langle \sin t, 2 \sin t, \sqrt{5} \cos t \rangle$, find the unit tangent vector, the principle unit normal vector and the equation of the osculating plane(the plane containing the unit tangent vector and the principle unit normal vector).

8. (8 points) Find the arc length of the curve $\vec{r}(t) = \left\langle t^2, \frac{4\sqrt{2}}{3} t^{3/2}, 2t \right\rangle$ for $1 \le t \le 3$.

9. (12 points) Given $\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t) = \langle \sin 3t, t^2, \cos 3t \rangle$ and $\vec{v}(0) = \langle 0, 0, 0 \rangle$, and $\vec{r}0 = \langle 2, -5, 3 \rangle$, find the position vector $\vec{r}(t)$.

10. Let P be a point with coordinates $(1, 2, -3)$, $\mathcal{L}$ be the line with equation $\vec{r}(t) = \langle 2 + 5t, -3 + t, -4t \rangle$ and $\mathcal{P}$ be the plane with equation x-3y+4z=5. Find the following:

    a. (4 points) the distance from P to $\mathcal{P}$.

    b. (4 points) the point at which $\mathcal{L}$ intersects $\mathcal{P}$.

    c. (4 points) than angle at which $\mathcal{L}$ intersects $\mathcal{P}$

11. (4 points (bonus)) Let $\vec{v} = \langle a, b, c \rangle$ and $\vec{w} = \langle \alpha, \beta, \gamma \rangle$. Give conditions on the constants a, b, c, α, β, and γ that guarantees that

    a. $\vec{v}$ is parallel to $\vec{w}$

    b. $\vec{v}$ is perpendicular to $\vec{w}$