Exam 1 **Name**____________________________________

Math 231-03 **Calculus III** **September 14, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 65 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.
- $\mathbf{u} \cdot \mathbf{v} = ||\mathbf{u}||||\mathbf{v}|| \cos\theta$ and $||\mathbf{u} \times \mathbf{v}|| = ||\mathbf{u}||||\mathbf{v}|| \sin\theta$
- $\kappa(t) = \dfrac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} = \dfrac{1}{|\mathbf{v}|}\left|\dfrac{d\mathbf{T}}{dt}\right|$

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1. (8 points) To be complete once exams are graded and returned. Please correct any problem with points deducted. All corrections should be completed neatly on a separate sheet of paper. Once you have finished your corrections, take your exam and corrections to the Math Department Help Room, Weir 220, and a tutor will check your answers and sign below. The checked solutions should be given to your instructor.

   Signature: ________________________________
   Print Name: ________________________________
   Date: ________________

Question	Points	Score
1	8	
2	20	
3	4	
4	12	
5	8	
6	8	
7	8	
8	16	
9	8	
10	8	
Total:	100	

2. Let $\mathbf{u} = \langle 3, -5, 2\rangle$ and $\mathbf{v} = \langle -9, 5, 1\rangle$, find

   a. (4 points) Find $\mathbf{v} \cdot \mathbf{u}$.

   b. (4 points) Find the angle between the vectors, $\mathbf{u}$ and $\mathbf{v}$.

   c. (6 points) Find two unit vectors that are orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

   d. (6 points) The projection of $\mathbf{u}$ onto $\mathbf{v}$.

3. (4 points) Find the values of x such that the vectors $\langle 3, 2, x\rangle$ and $\langle 2x, 4, x\rangle$ are orthogonal.

4. (12 points) Find a line that is perpendicular to the lines $\mathbf{r} = \langle 4t, 1+2t, 3t\rangle$ and $\mathbf{R} = \langle -1+s, -7+2s, -12+3s\rangle$ and passes through the point of intersection of the lines $\mathbf{r}$ and $\mathbf{R}$.

5. (8 points) Find the area of the parallelogram that has two adjacent sides $\mathbf{u} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

6. (8 points) Find a parametric equation of the line tangent to the curve $\mathbf{r}(t) = \left\langle \sqrt{2t+1}, \sin \pi t, 4\right\rangle$ at $t = 4$.

7. (8 points) If the velocity of a particle is $\mathbf{v}(t) = \left\langle \frac{t}{t^2+1}, te^{-t^2}, -\frac{2t}{\sqrt{t^2+4}} \right\rangle$, find the position vector $\vec{r}(t)$ if $\mathbf{r}(0) = \left\langle 1, \frac{3}{2}, -3 \right\rangle$.

8. (16 points) For the vector curve $\mathbf{r}(t) = \langle \cos t, \sin t, \ln \cos t \rangle$, find the unit tangent vector, the principle unit normal vector, the binormal vector and curvature **all at** $t = 0$.

9. (8 points) Find the arc length of the curve $\mathbf{r}(t) = \left\langle \cos(2t), 2t^{3/2}, \sin(2t) \right\rangle$ for $0 \leq t \leq 5$.

10. (8 points) Determine if the curve $\mathbf{r}(t) = \langle 13\cos t, 5\sin t, 12\sin t \rangle$ for $0 \leq t \leq \pi$ uses arc length as a parameter. If not, find a description that uses arc length as a parameter.