Practice Questions for Exam 3 **Math 132**

1. Determine whether or not the sequence $\{a_k\}$ converges and find its limit if it does converge.

   a. $a_k = \dfrac{8k-7}{7k-8}$

   b. $a_k = \dfrac{k-e^k}{k+e^k}$

   c. $a_k = \left(1+\dfrac{1}{k}\right)^k$

   d. $a_k = \dfrac{k^3}{10k^2+1}$

2. Find the Taylor Series for

   a. $f(x) = \dfrac{1}{(x-4)^2}$ at $x_0 = 5$.

   b. $f(x) = \sin x$ at $x_0 = \dfrac{\pi}{2}$

3. Give that $e^x = \sum_{k=0}^{\infty} \dfrac{x^k}{k!}$ and $\sin x = \sum_{k=0}^{\infty} \dfrac{(-1)^k x^{2k+1}}{(2k+1)!}$, find the power series for each of the following:

   a. $g(x) = x^2 e^x$

   b. $g'(x)$

   c. $\int \dfrac{\sin x}{x}\,dx$

   d. $\int \dfrac{e^x - 1}{x}\,dx$

4. Find the sum of the following series:

   a. $\sum_{k=1}^{\infty} \left(\dfrac{e}{\pi}\right)^k$

   b. $\sum_{n=2}^{\infty} \dfrac{2}{n^2-1}$

   c. $\sum_{k=1}^{\infty} \left[\left(\dfrac{7}{11}\right)^k - \left(\dfrac{3}{5}\right)^{k+1}\right]$

5. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. . Justify your answers by citing relevant tests or reason.

   a. $\sum_{k=2}^{\infty} \dfrac{(-1)^k \sqrt{k}}{\ln k}$

b. $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^{1/3}}$

c. $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$

6. Determine the interval of convergence for the following power series:

a. $\sum_{k=0}^{\infty} \frac{k!x^{2k}}{10^k}$

b. $\sum_{k=1}^{\infty} \frac{(x-1)^k}{k3^k}$

c. $\sum_{k=0}^{\infty} \frac{(2x-1)^k}{k^2+1}$

7. Determine whether the following series converge or diverge. Justify your answers by citing relevant tests or reason.

a. $\sum_{k=0}^{\infty} \frac{(-2)^k}{3^k+1}$

b. $\sum_{k=0}^{\infty} \frac{k!}{e^{k^2}}$

c. $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

d. $\sum_{k=2}^{\infty} \frac{(-1)^k}{k \ln k}$

e. $\sum_{j=2}^{\infty} \frac{1}{j\sqrt{\ln j}}$

f. $\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n}\right)^n$

g. $\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{k\sqrt{k}}$

h. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+25}$

Topics:

- ✓ Series and Sequences:
  - Does the sequence converge or diverge? Note: Can you use any of the tests for series?
  - What is the *nth* partial sum? While, it would not be possible to find the 10^{th} partial sum it is possible to write it out.
  - Geometric series and telescoping series, find the sum.
  - Test various types of series for convergence. The tests on the next page will be provided. Be sure you check the conditions of the test before applying it. Also be sure you state what test you use in your conclusion.

- Is the series absolutely convergent, conditionally convergent or divergent?

✓ Power Series:
  - Use the ratio test (with absolute values) to find interval of convergence and radius of convergence.
  - Find a Taylor polynomial or Taylor series.
  - Manipulate a known series.
  - Find the derivative or integral of a series.

For additional problems, check out the review problems for Chapters 7 and 8. Note the questions above are simply a sample of questions possible for the exam; it is possible that other types of questions may appear on your exam.

Test	Conditions	Conclusion
POSITIVE TERM TESTS		
Integral Test $a_k = f(k)$, $a(x)$ is positive, continuous and decreasing	$\int_1^\infty a(x)dx$ diverges	Diverges
	$\int_1^\infty a(x)dx$ converges	Converges
Comparison Test	$a_k \ge b_k \ge 0$ and $\sum_{k=1}^\infty b_k$ diverges	Diverges
	$0 \le a_k \le b_k$ and $\sum_{k=1}^\infty b_k$ converges	Converges
Limit Comparison test $\lim_{k\to\infty} \frac{a_k}{b_k} = L, \quad 0 < L < \infty$	$\sum_{k=1}^\infty b_k$ diverges	Diverges
	$\sum_{k=1}^\infty b_k$ converges	Converges
Ratio Test $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \rho$	$\rho > 1$	Diverges
	$\rho < 1$	Converges
	$\rho = 1$	Inconclusive
Root Test $\lim_{k\to\infty} \sqrt[k]{a_k} = \rho$	$\rho > 1$	Diverges
	$\rho < 1$	Converges
	$\rho = 1$	Inconclusive
Series with some/all non-positive terms		
Alternating Series Test $\sum_{k=1}^\infty (-1)^k a_k$ or $\sum_{k=1}^\infty (-1)^{k-1} a_k$	$\lim_{k\to\infty} a_k = 0$ and sequence a_k is decreasing $(a_k \ge a_{k+1}, \forall k)$	Converges
	If $\lim_{k\to\infty} a_k \ne 0$, use Divergence Test	