Math 132 Calculus II **Practice Exam 2**

Evaluate the following integrals using the appropriate technique.

1. **Basic techniques**

   a. $\int (9x-2)^{-3}\,dx$

   b. $\int \frac{1}{x^2-2x+10}\,dx$

   c. $\int \frac{e^x}{e^x-e^{-x}}\,dx$

   d. $\int \frac{1}{\sqrt{27-6x-x^2}}\,dx$

   e. $\int_2^4 \frac{x^2+2}{x-1}\,dx$

2. **Integration by Parts**

   a. $\int xe^x\,dx$

   b. $\int \frac{x}{\sqrt{x+1}}\,dx$

   c. $\int x^2\ln x\,dx$

   d. $\int x\tan^{-1}(x)\,dx$

   e. $\int e^{3x}\cos(2x)\,dx$

   f. $\int x^2e^{4x}\,dx$

   g. $\int_0^{\ln 2} x\cosh(2x)\,dx$

   h. $\int_1^e \ln(2x)\,dx$

3. **Trig. Integrals**

   a. $\int \sin^3 x\,dx$

   b. $\int \sin^3 x\cos^2 x\,dx$

   c. $\int \sin^2 x\cos^4 x\,dx$

   d. $\int \tan^3 x\,dx$

   e. $\int \tan^{-5} x\sec^2 x\,dx$

   f. $\int \tan x\sec^3 x\,dx$

   g. $\int_0^{\pi/4} \sec^4 x\,dx$

4. **Trig. Substitutions**

   a. $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\,dx$

   b. $\int \sqrt{36-x^2}\,dx$

   c. $\int \frac{1}{x^2\sqrt{x^2+9}}\,dx$

   d. $\int \frac{1}{\sqrt{16+4x^2}}\,dx$

   e. $\int \frac{\sqrt{x^2-4}}{x}\,dx$

   f. $\int \frac{1}{(x^2-25)^{3/2}}\,dx$

①

a. $\int (9x-2)^{-3}\,dx$

$u = 9x-2 \quad du = 9\,dx$

$$= \frac{1}{9}\int (u)^{-3}\,du = \frac{1}{9}\,\frac{u^{-2}}{-2} + C$$

$$= \boxed{-\frac{1}{18}(9x-2)^{-2} + C}$$

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b. $\int \frac{1}{x^2-2x+10}\,dx$

Complete the square

$x^2 - 2x + 1 - 1 + 10$

$= (x-1)^2 + 9$

$$= \int \frac{1}{(x-1)^2+9}\,dx = \boxed{\frac{1}{3}\arctan\left(\frac{x-1}{3}\right) + C}$$

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c. $\int \frac{e^x}{e^x - e^{-x}}\,dx$

$$= \int \frac{e^x}{e^x - \frac{1}{e^x}}\,dx$$

$$= \int \frac{e^x}{\frac{e^{2x}-1}{e^x}}\,dx = \int \frac{e^{2x}}{e^{2x}-1}\,dx$$

$u = e^{2x}-1$

$du = 2e^{2x}\,dx$

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|e^{2x}-1| + C}$$

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d. $\int \frac{1}{\sqrt{27-6x-x^2}}\,dx$

Complete the square

$-(x^2+6x+9-9)+27$

$-(x+3)^2+9+27$

$36-(x+3)^2$

$$= \int \frac{1}{\sqrt{36-(x+3)^2}}\,dx = \boxed{\arcsin\left(\frac{x+3}{6}\right) + C}$$

e. $\int_2^4 \frac{x^2+2}{x-1}\,dx$

$$\begin{array}{r} x+1 \\ x-1\ \overline{\big)\, x^2+0x+2} \\ -(x^2-x) \\ \hline x+2 \\ -(x-1) \\ \hline 3 \end{array}$$

$$\int_2^4 \frac{x^2+2}{x-1}\,dx = \int_2^4 \left(x+1+\frac{3}{x-1}\right)dx$$

$$= \frac{x^2}{2}\Big|_2^4 + x\Big|_2^4 + 3\ln|x-1|\Big|_2^4$$

$$= \frac{1}{2}(16-4) + (4-2) + 3(\ln 3 - \ln 1) = \boxed{8+3\ln 3}$$

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2. a. $\int xe^x\,dx$

$u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

$$\int xe^x\,dx = xe^x - \int e^x\,dx$$

$$= \boxed{xe^x - e^x + C}$$

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b. $\int \frac{x}{\sqrt{x+1}}\,dx$

$u = x \qquad dv = \frac{1}{\sqrt{x+1}}\,dx$

$du = dx \qquad v = 2\sqrt{x+1}$

$$\int \frac{x}{\sqrt{x+1}}\,dx = 2x\sqrt{x+1} - 2\int \sqrt{x+1}\,dx$$

$$= \boxed{2x\sqrt{x+1} - \frac{4}{3}(x+1)^{3/2} + C}$$

c. $\int x^2 \ln x \, dx$

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x} dx \qquad v = \frac{1}{3}x^3$

$$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$$

$$= \boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$$

d. $\int x \tan^{-1}(x) \, dx$

$u = \tan^{-1} x \qquad dv = x\,dx$

$du = \frac{1}{1+x^2} dx \qquad v = \frac{1}{2}x^2$

$$\int x \tan^{-1} x \, dx = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2} dx$$

$$= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\int \left(\frac{x^2+1}{1+x^2} + \frac{-1}{1+x^2}\right) dx$$

$$= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{1}{1+x^2} dx$$

$$= \boxed{\frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}x + \frac{1}{2}\arctan x + C}$$

e. $\int e^{3x} \cos(2x) \, dx = I$

$u = e^{3x} \qquad dv = \cos(2x) dx$

$du = 3e^{3x} dx \qquad v = \frac{1}{2}\sin(2x)$

$$I = \frac{1}{2}e^{3x}\sin(2x) - \frac{3}{2}\int e^{3x} \sin 2x \, dx$$

$u = e^{3x} \qquad dv = \sin 2x$

$du = 3e^{3x} dx \qquad v = -\frac{1}{2}\cos 2x$

$$I = \frac{1}{2}e^{3x}\sin(2x) - \frac{3}{2}\left(-\frac{1}{2}e^{3x}\cos 2x + \int \frac{3}{2}e^{3x}\cos 2x\right)$$

$$I = \frac{1}{2}e^{3x}\sin 2x + \frac{3}{4}e^{3x}\cos 2x - \frac{9}{4} I$$

$$\frac{13}{4} I = \frac{1}{2}e^{3x}\sin 2x + \frac{3}{4}e^{3x}\cos 2x \Rightarrow I = \boxed{\frac{2}{13}e^{3x}\sin 2x + \frac{3}{13}e^{3x}\cos 2x + C}$$

f. $\int x^2 e^{4x}\,dx$

$u = x^2 \quad dv = e^{4x}dx$
$du = 2x\,dx \quad v = \frac{1}{4}e^{4x}$

$$\int x^2 e^{4x}dx = \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\int x e^{4x}dx$$

$u = x \quad dv = e^{4x}dx$
$du = dx \quad v = \frac{1}{4}e^{4x}$

$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\left(\frac{1}{4}xe^{4x} - \frac{1}{4}\int e^{4x}dx\right)$$

$$= \boxed{\frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{8}\cdot\frac{1}{4}e^{4x} + C}$$

g. $\int_0^{\ln 2} x\cosh(2x)\,dx$

$u = x \quad dv = \cosh(2x)dx$
$du = dx \quad v = \frac{1}{2}\sinh(2x)$

$$= \frac{1}{2}x\sinh(2x)\Big|_0^{\ln 2} - \frac{1}{2}\int_0^{\ln 2}\sinh(2x)dx$$

$$= \frac{1}{2}x\sinh(2x)\Big|_0^{\ln 2} - \frac{1}{2}\cdot\frac{1}{2}\cosh(2x)\Big|_0^{\ln 2}$$

$$= \frac{1}{2}\ln 2\sinh(2\ln 2) - 0 - \frac{1}{4}\cosh(2\ln 2) + \frac{1}{4}\cosh(0)$$

$$= \frac{1}{2}\ln 2\left(\frac{e^{\ln 2^2} - e^{\ln 2^{-2}}}{2}\right) - \frac{1}{4}\left(\frac{e^{\ln 2^2} + e^{\ln 2^{-2}}}{2}\right) + \frac{1}{4}\left(\frac{e^0 + e^0}{2}\right)$$

$$\boxed{= \frac{1}{4}\ln 2\left[4 - \frac{1}{4}\right] - \frac{1}{8}\left(4 + \frac{1}{4}\right) + \frac{1}{4}}$$

h. $\int_1^e \ln(2x)\,dx$

$u = \ln(2x)$ $\quad dv = dx$

$du = \frac{1}{2x}\cdot 2dx$ $\quad v = x$

$$\int_1^e \ln(2x)dx = x\ln(2x)\Big|_1^e - \int_1^e dx$$

$$= x\ln(2x)\Big|_1^e - x\Big|_1^e$$

$$= \boxed{e\ln(2e) - \ln 2 - e + 1}$$

3

a. $\int \sin^3 x\,dx$

$$= \int \sin^2 x \cdot \sin x\,dx = \int (1-\cos^2 x)\sin x\,dx$$

$u = \cos x$ $\quad du = -\sin x\,dx$

$$= -\int (1-u^2)du = \int (u^2-1)du$$

$$= \frac{1}{3}u^3 - u + C = \boxed{\frac{1}{3}\cos^3 x - \cos x + C}$$

b. $\int \sin^3 x \cos^2 x\,dx$ $= \int \sin^2 x \cos^2 x \cdot \sin x\,dx$

$= \int (1-\cos^2 x)\cos^2 x \sin x\,dx$ $\quad$ let $u = \cos x$ $du = -\sin x dx$

$$= -\int (1-u^2)u^2\,du = \int (u^4 - u^2)du$$

$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C = \boxed{\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C}$$

c. $\int \sin^2 x \cos^4 x \, dx$ $= \int \sin^2 x \cos^2 x \cos^2 x \, dx$

$$= \int \frac{1}{2}(1-\cos 2x)\,\frac{1}{2}(1+\cos 2x)\,\frac{1}{2}(1+\cos 2x)\,dx$$

$$= \frac{1}{8}\int (1-\cos^2 2x)(1+\cos 2x)\,dx$$

$$= \frac{1}{8}\int (1-\cos^2 2x+\cos 2x-\cos^3 2x)\,dx$$

$$= \frac{1}{8}\left[\int dx - \int \cos^2 2x\,dx + \int \cos 2x\,dx - \int \cos^3 2x\,dx\right]$$

$$= \frac{1}{8}\left[x - \frac{1}{2}\int (1+\cos 4x)\,dx + \frac{1}{2}\sin 2x - \int (1-\sin^2 2x)\cos 2x\,dx\right]$$

$u = \sin 2x$

$du = 2\cos 2x\,dx$

$$= \frac{1}{8}x - \frac{1}{16}\left(x+\frac{1}{4}\sin 4x\right) + \frac{1}{16}\sin 2x - \frac{1}{8}\cdot\frac{1}{2}\int (1-u^2)\,du$$

$$\boxed{= \frac{1}{16}x - \frac{1}{64}\sin 4x + \frac{1}{16}\sin 2x - \frac{1}{16}\sin 2x + \frac{1}{16}\cdot\frac{1}{3}\sin^3 2x + C}$$

d. $\int \tan^3 x \, dx$ $= \int \tan^2 x \cdot \tan x \, dx$

$$= \int (\sec^2 x - 1)\tan x\,dx = \int \tan x \sec^2 x\,dx - \int \tan x\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$$\boxed{= \frac{1}{2}\tan^2 x - \ln|\sec x| + C}$$

e. $\int \tan^{-5} x \sec^2 x \, dx = \int \frac{1}{\tan^5 x} \sec^2 x \, dx$

$$= \int \frac{\cos^5 x}{\sin^5 x} \cdot \frac{1}{\cos^2 x} \, dx = \int \frac{\cos^3 x}{\sin^5 x} \, dx$$

$$= \int \frac{\cos^2 x \cdot \cos x}{\sin^5 x} \, dx = \int \frac{(1-\sin^2 x)\cos x}{\sin^5 x} \, dx$$

$u = \sin x \quad du = \cos x \, dx$

$$= \int \frac{1-u^2}{u^5} \, du = \int (u^{-5} - u^{-3}) \, du$$

$$= \frac{u^{-4}}{-4} - \frac{u^{-2}}{-2} + C = \boxed{-\frac{1}{4}\frac{1}{\sin^4 x} + \frac{1}{2}\frac{1}{\sin^2 x} + C}$$

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f. $\int \tan x \sec^3 x \, dx = \int \tan x \cdot \sec x \cdot \sec^2 x \, dx$

$u = \sec x \quad du = \sec x \tan x \, dx$

$$= \int u^2 \, du = \frac{1}{3}u^3 + C = \boxed{\frac{1}{3}\sec^3 x + C}$$

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g. $\int_0^{\pi/4} \sec^4 x \, dx = \int_0^{\pi/4} \sec^2 x \cdot \sec^2 x \, dx$

$$= \int_0^{\pi/4} (1 + \tan^2 x)\sec^2 x \, du \qquad u = \tan x \quad du = \sec^2 x \, dx$$

$$= \int_0^1 (1 + u^2) \, du = \left(u + \frac{1}{3}u^3\right)\Big|_0^1 = 1 + \frac{1}{3} = \boxed{\frac{4}{3}}$$

④

a. $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\,dx$

$4\cos^2\theta = 4 - 4\sin^2\theta$
$= 4 - x^2$

$x = 2\sin\theta \quad dx = 2\cos\theta\, d\theta$

$0 = 2\sin\theta \quad \frac{\sqrt{2}}{2} = \sin\theta$
$0 = \sin\theta \quad \theta = \pi/4$
$\theta = 0$

$$= \int_0^{\pi/4} \frac{4\sin^2\theta \cdot 2\cos\theta}{\sqrt{4\cos^2\theta}}\,d\theta$$

$$= \int_0^{\pi/4} 4\sin^2\theta\, d\theta = 4\cdot\frac{1}{2}\int_0^{\pi/4}(1-\cos 2\theta)\,d\theta$$

$$= 2\left[\theta - \frac{1}{2}\sin 2\theta\right]\Big|_0^{\pi/4} = 2\cdot\frac{\pi}{4} - \sin\frac{\pi}{2}$$

$$= \boxed{\pi/2 - 1}$$

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b. $\int \sqrt{36-x^2}\,dx$

$36\cos^2\theta = 36 - 36\sin^2\theta$
$= 36 - x^2$

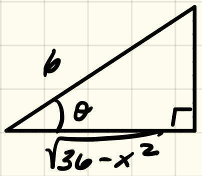
$x = 6\sin\theta \quad dx = 6\cos\theta\, d\theta$

$$= \int \sqrt{36\cos^2\theta} \cdot 6\cos\theta\, d\theta$$

$$= 36\int \cos^2\theta\, d\theta = 36\cdot\frac{1}{2}\int(1-\cos 2\theta)\,d\theta$$

$$= 18\left[\theta - \frac{1}{2}\sin 2\theta\right] + C = 18\theta - 18\sin\theta\cos\theta + C$$

$$= \boxed{18\arcsin\left(\frac{x}{6}\right) - 18\left(\frac{x}{6}\right)\left(\frac{\sqrt{36-x^2}}{6}\right) + C}$$



$\frac{x}{6} = \sin\theta$

c. $\displaystyle\int \frac{1}{x^2\sqrt{x^2+9}}\,dx$

$9\sec^2\theta = 9\tan^2\theta + 9$

$= x^2 + 9$

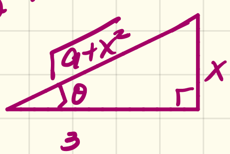
$x = 3\tan\theta \qquad dx = 3\sec^2\theta\, d\theta$

$$\int \frac{1}{x^2\sqrt{x^2+9}}\,dx = \int \frac{1}{9\tan^2\theta\sqrt{9\sec^2\theta}} \cdot 3\sec^2\theta\, d\theta$$

$$= \frac{1}{9}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{9}\int \frac{\cos^2\theta}{\sin^2\theta}\cdot\frac{1}{\cos\theta}\,d\theta = \frac{1}{9}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta \quad du = \cos\theta\, d\theta$

$$= \frac{1}{9}\int \frac{1}{u^2}\,du = \frac{1}{9}\left(-\frac{1}{u}\right) + C = -\frac{1}{9}\,\frac{1}{\sin\theta} + C$$



$$= \boxed{-\frac{1}{9}\cdot\frac{\sqrt{9+x^2}}{x} + C}$$

d. $\displaystyle\int \frac{1}{\sqrt{16+4x^2}}\,dx$

$16\sec^2\theta = 16 + 16\tan^2\theta$

$= 16 + 4x^2$

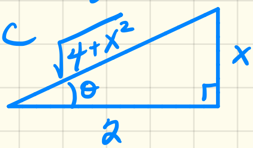
$2x = 4\tan\theta$

$x = 2\tan\theta \qquad dx = 2\sec^2\theta\, d\theta$

$$= \int \frac{1}{\sqrt{16\sec^2\theta}} \cdot 2\sec^2\theta\, d\theta$$

$\frac{x}{2} = \tan\theta$

$$= \frac{1}{2}\int \sec\theta\, d\theta = \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$



$$= \boxed{\frac{1}{2}\ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C}$$

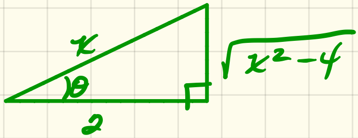
e. $\int \frac{\sqrt{x^2-4}}{x}\,dx$

$4\tan^2\theta = 4\sec^2\theta - 4$

$= x^2 - 4$

$x = 2\sec\theta \quad dx = 2\sec\theta\tan\theta\,d\theta$

$\frac{x}{2} = \sec\theta$



$$= \int \frac{\sqrt{4\tan^2\theta}\cdot 2\sec\theta\tan\theta\,d\theta}{2\sec\theta\,d\theta}$$

$$= \int 2\tan^2\theta\,d\theta$$

$$= 2\int (\sec^2\theta - 1)\,d\theta$$

$$= 2\left[\tan\theta - \theta\right] + C = 2\tan\theta - 2\theta + C$$

$$= \boxed{2\left(\frac{\sqrt{x^2-4}}{2}\right) - 2\arctan\left(\frac{\sqrt{x^2-4}}{2}\right)}$$

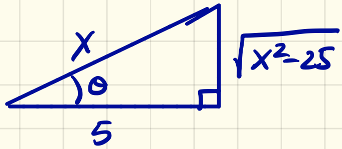
f. $\int \frac{1}{(x^2-25)^{3/2}}\,dx$

$25\tan^2\theta = 25\sec^2\theta - 25$

$= x^2 - 25$

$x = 5\sec\theta \quad dx = 5\sec\theta\tan\theta\,d\theta$

$\frac{x}{5} = \sec\theta$



$$= \int \frac{1}{(25\tan^2\theta)^{3/2}} \cdot 5\sec\theta\tan\theta\,d\theta$$

$$= \frac{5}{5^3}\int \frac{\sec\theta\tan\theta}{\tan^3\theta}\,d\theta = \frac{1}{25}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{25}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

$$= \frac{1}{25}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta = \frac{1}{25}\int \frac{1}{u^2}\,du = -\frac{1}{25}\,\frac{1}{\sin\theta} + C$$

$$= \boxed{-\frac{1}{25}\left(\frac{x}{\sqrt{x^2-25}}\right) + C}$$

5. **Partial Fractions**

a. $\displaystyle\int \frac{10x}{x^2-2x-24}\,dx$

b. $\displaystyle\int \frac{21x^2}{x^3-x^2-12x}\,dx$

c. $\displaystyle\int \frac{2}{x^3+x^2}\,dx$

d. $\displaystyle\int \frac{x^2-4}{x^3-2x^2+x}\,dx$

e. $\displaystyle\int \frac{x^2+3x+2}{x(x^2+2x+2)}\,dx$

f. $\displaystyle\int \frac{x+1}{x(x^2+4)}\,dx$

6. **Improper Integrals**

a. $\displaystyle\int_0^\infty \frac{1}{(x+1)^3}\,dx$

b. $\displaystyle\int_{-\infty}^1 2^x\,dx$

c. $\displaystyle\int_2^\infty \frac{1}{\sqrt{x}}\,dx$

d. $\displaystyle\int_{-3}^1 \frac{1}{(2x+6)^{2/3}}\,dx$

e. $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

f. $\displaystyle\int_{-1}^1 \ln x^2\,dx$

7. **Differential Equations**
Find the solution to the following differential equations. If an initial condition is given, determine the arbitarary constant in the solution. Otherwise, find the general solution.

a. $y'(x) = -2y-4$

b. $e^{-t}\dfrac{dy}{dt} = \dfrac{1}{2y}, \quad y(\ln 2) = 1$

c. $y'(t) = \dfrac{e^y}{t}$

d. $y'(t) = y(4t^3+1), \quad y(0) = 2$

For additional problems, check out the review problems for Chapter 7. Note the questions above are simply a sample of questions possible for the exam; it is possible that other types of questions may appear on your exam.

. $\displaystyle\int \frac{10x}{x^2-2x-24}\,dx$

① Deg Num (1) < Deg Den (2)

② $x^2-2x-24 = (x-6)(x+4)$

③ $\dfrac{10x}{(x-6)(x+4)} = \dfrac{A}{x-6} + \dfrac{B}{x+4} \Rightarrow 10x = A(x+4) + B(x-6)$

let $x = 6 \Rightarrow 60 = A(10) \Rightarrow A = 6$

$x = -4 \qquad -40 = B(-10) \qquad B = 4$

$$\int \frac{10x}{x^2-2x-24}\,dx = \int\left(\frac{6}{x-6} + \frac{4}{x+4}\right)dx$$

$$= \boxed{6\ln|x-6| + 4\ln|x+4| + C}$$

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b. $\displaystyle\int \frac{21x^2}{x^3-x^2-12x}\,dx = \int \frac{21x^2}{x(x^2-x-12)}\,dx = \int \frac{21x}{x^2-x-12}$

① Deg Num (1) < deg den (2)

② $x^2-x-12 = (x-4)(x+3)$

③ $\dfrac{21x}{(x-4)(x+3)} = \dfrac{A}{x-4} + \dfrac{B}{x+3}$

$\Rightarrow 21x = A(x+3) + B(x-4)$

$x = 4 \qquad 84 = A(7) \qquad A = 12$

$x = -3 \qquad -63 = B(-7) \qquad A = 9$

$$\int \frac{21x}{(x-4)(x+3)}\,dx = \int\left(\frac{12}{x-4} + \frac{9}{x+3}\right)dx = \boxed{12\ln|x-4| + 9\ln|x+3| + C}$$

c. $\displaystyle\int \frac{2}{x^3+x^2}\,dx$

① Deg Num (0) < deg den (2)

② $x^3+x^2 = x^2(x+1)$

③ $$\frac{2}{x^2(x+1)} = \frac{Ax+B}{x^2} + \frac{C}{x+1}$$

$\Rightarrow 2 = (Ax+B)(x+1) + Cx^2$

$x=-1 \qquad 2 = C$

$x=0 \qquad 2 = (0+B)(1) + 0 \Rightarrow 2 = B$

$x=1 \qquad 2 = (A+B)(2) + C$

$\qquad 2 = 2A + 4 + 2$

$\qquad -4 = 2A \qquad A = -2$

$$\int \frac{2}{x^3+x^2}\,dx = \int\left(\frac{-2x+2}{x^2} + \frac{2}{x+1}\right)dx$$

$$= \int\left(-\frac{2}{x} + \frac{2}{x^2} + \frac{2}{x+1}\right)dx = \boxed{-2\ln|x| - \frac{2}{x} + 2\ln|x+1| + C}$$

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d. $\displaystyle\int \frac{x^2-4}{x^3-2x^2+x}\,dx$

① deg Num (2) < deg den (3)

② $x^3-2x^2+x = x(x^2-2x+1) = x(x-1)^2$

③ $$\frac{x^2-4}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$\Rightarrow x^2-4 = A(x-1)^2 + Bx(x-1) + Cx$

$x=0 \qquad -4 = A$

$x=1 \qquad -3 = C$

$x=-1 \qquad -3 = 4A + 2B - C$

$\qquad -3 = -16 + 2B + 3 \Rightarrow 10 = 2B \Rightarrow B = 5$

Next →

$$\int\left(\frac{-4}{x}+\frac{5}{x-1}+\frac{-3}{(x-1)^2}\right)dx$$

$$=\boxed{-4\ln|x|+5\ln|x-1|+\frac{3}{x-1}+C}$$

---

e. $\displaystyle\int \frac{x^2+3x+2}{x(x^2+2x+2)}\,dx$

① deg num (2) < deg den (3) → 3 coef to find

② $x(x^2+2x+2)$ ← 2 fractions

Note x^2+2x+2 is irreducible

$b^2-4ac<0$ $(4-8<0)$

③ $$\frac{x^2+3x+2}{x(x^2+2x+2)}=\frac{A}{x}+\frac{Bx+C}{x^2+2x+2}$$

$\Rightarrow x^2+3x+2 = A(x^2+2x+2)+(Bx+C)x$

$x=0 \Rightarrow 2=A(2) \Rightarrow A=1$

$x=1 \quad 6=(1)(5)+B+C \Rightarrow B+C=-1$

$x=-1 \quad 0=1+B-C \Rightarrow B-C=-1$

$2B=-2 \rightarrow B=-1$

so $-1+C=-1 \Rightarrow C=0$

$$\int\left(\frac{1}{x}+\frac{-x}{x^2+2x+2}\right)dx$$

→ Note: $x^2+2x+1-1+2$
$=(x+1)^2+1$

$$=\int\frac{1}{x}dx+\int\frac{-x-1+1}{(x+1)^2+1}dx=\int\frac{1}{x}dx-\int\frac{x+1}{(x+1)^2+1}dx+\int\frac{1}{(x+1)^2+1}dx$$

$u=(x+1)^2+1$
$du=2(x+1)dx$

$$=\boxed{\ln|x|-\frac{1}{2}\ln\left|(x+1)^2+1\right|+\arctan(x+1)+C}$$

f. $\displaystyle\int \frac{x+1}{x(x^2+4)}\,dx$

① deg num(1) < deg den(3) ← 3 coef to find
② $x(x^2+4)$ ← 2 fractions

③ $$\frac{x+1}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$\Rightarrow x+1 = A(x^2+4) + (Bx+C)x$$

$x=0 \quad 1 = 4A \Rightarrow A = 1/4$

$x=1 \quad 2 = \frac{1}{4}(5) + B + C \Rightarrow B + C = 3/4$

$x=-1 \quad 0 = \frac{1}{4}(5) + B - C \Rightarrow B - C = -5/4$

$2B = -2/4$

$B = -1/4$

$-\frac{1}{4} + C = \frac{3}{4} \quad C = 1$

$$\int \left(\frac{1/4}{x} + \frac{-1/4\,x + 1}{x^2+4} \right) dx = \frac{1}{4}\int \frac{1}{x}\,dx - \frac{1}{4}\int \frac{x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx$$

$$\boxed{= \frac{1}{4}\ln|x| - \frac{1}{4}\cdot\frac{1}{2}\ln|x^2+4| + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C}$$

---

⑥ a. $\displaystyle\int_0^\infty \frac{1}{(x+1)^3}\,dx = \lim_{c\to\infty} \int_0^c \frac{1}{(x+1)^3}\,dx$

$$= \lim_{c\to\infty} -\frac{1}{2}\cdot\frac{1}{(x+1)^2}\Big|_0^c = \lim_{c\to\infty}\left[\frac{-1}{2(c+1)^2} + \frac{1}{2}\right]$$

$= \frac{1}{2}$ $\quad\therefore$ Integral converges to 2

---

b. $\int_{-\infty}^{1} 2^x \, dx = \lim_{c \to -\infty} \int_c^1 e^{x \ln 2} dx$

$$= \lim_{c \to -\infty} \frac{1}{\ln 2} e^{x \ln 2} \Big|_c^1 = \frac{1}{\ln 2} \cdot \lim_{c \to -\infty} \left[e^{\ln 2} - e^{c \ln 2} \right]$$

$$= \frac{1}{\ln 2} \cdot e^{\ln 2} = \frac{2}{\ln 2}$$

$\boxed{\therefore \text{ integral converges to } 2/\ln 2}$

c. $\int_{2}^{\infty} \frac{1}{\sqrt{x}} \, dx = \lim_{c \to \infty} \int_2^c \frac{1}{\sqrt{x}} dx = \lim_{c \to \infty} 2\sqrt{x} \Big|_2^c$

$$= \lim_{c \to \infty} \left[2\sqrt{c} - 2\sqrt{2} \right] = \infty$$

$\boxed{\therefore \text{ integral diverges}}$

d. $\int_{-3}^{1} \frac{1}{(2x+6)^{2/3}} \, dx = \lim_{c \to -3^+} \int_c^1 (2x+6)^{-2/3} dx$

$(2x+6=0 \text{ at } x=-3)$

$$= \lim_{c \to -3^+} \frac{1}{2} \cdot 3 \, (2x+6)^{1/3} \Big|_c^1$$

$$= \lim_{c \to -3^+} \frac{3}{2} \left[8^{1/3} - (2c+6)^{1/3} \right] = \frac{3}{2} \cdot 8^{1/3} = \boxed{3}$$

$\boxed{\therefore \text{ integral conv to } 3}$

e. $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{c\to 0^+} \int_c^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$= \lim_{c\to 0^+} \left[2e - 2e^{\sqrt{c}}\right] = 2e - 2$ $\boxed{\therefore \text{ integral conv to } 2e-2}$

Aside $\int_c^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int_{\sqrt{c}}^1 e^u = 2e^u\Big|_{\sqrt{c}}^1 = 2e - 2e^{\sqrt{c}}$

$u = \sqrt{x}$ $du = \frac{1}{2\sqrt{x}}dx$

---

f. $\int_{-1}^1 \ln x^2\,dx = \int_{-1}^0 \ln x^2\,dx + \int_0^1 \ln x^2\,dx = \boxed{4}$

$\boxed{\text{conv since both conv}}$

Now $\int_0^1 \ln x^2\,dx = \lim_{c\to 0^+} \int_c^1 \ln x^2\,dx = \lim_{c\to 0^+}(-c\ln c^2 - 2 + 2c)$

$= \lim_{c\to 0^+} \frac{\ln c^2}{1/c} - 2 \overset{L}{=} \lim_{c\to 0^+} \frac{2/c}{-1/c^2} - 2 = 0 - 2 = -2$

& $\int_{-1}^0 \ln x^2\,dx = \lim_{c\to 0^-} \int_{-1}^c \ln x^2 = \lim_{c\to 0^-}(c\ln c^2 - 2c - 2)$

$= \lim_{c\to 0^-} \frac{\ln c^2}{1/c} - 2 \overset{L}{=} \lim_{c\to 0^-} \frac{2/c}{-1/c^2} - 2 = -2$

Aside $\int_c^1 \ln x^2\,dx$

$u = \ln x^2$ $dv = dx$

$du = \frac{1}{x^2}\cdot 2x\,dx$ $v = x$

$= x\ln x^2\Big|_c^1 - \int_c^1 2\,dx$

$= x\ln x^2\Big|_c^1 - 2x\Big|_c^1 = 0 - c\ln c^2 - 2 + 2c$

7.

a. $y'(x) = -2y - 4$

$$\frac{dy}{dx} = -2y-4 \Rightarrow dy = (-2y-4)\,dx$$

$$\Rightarrow \frac{1}{-2y-4}\,dy = dx \Rightarrow \int \frac{1}{-2y-4}\,dy = \int dx$$

$$\Rightarrow -\frac{1}{2}\ln|-2y-4| = x + c$$

$$\Rightarrow \ln|-2y-4| = -2x + c \Rightarrow |-2y-4| = e^{-2x+c}$$

$$\Rightarrow 2y-4 = e^{c}e^{-2x} \Rightarrow 2y = Ce^{-2x}+4$$

$$\boxed{y = Ce^{-2x}+2}$$

---

b. $e^{-t}\dfrac{dy}{dt} = \dfrac{1}{2y}, \quad y(\ln 2) = 1$

$$e^{-t}\frac{dy}{dt} = \frac{1}{2y} \Rightarrow 2y\,dy = e^{t}\,dt$$

$$\int 2y\,dy = \int e^{t}\,dt \rightarrow y^2 = e^{t} + c$$

$$\Rightarrow 1 = e^{\ln 2} + c \Rightarrow 1 = 2 + c \Rightarrow c = -1$$

$$\boxed{y^2 = e^{t} - 1 \quad \text{or} \quad y = \pm\sqrt{e^{t}-1}}$$

---

c. $y'(t) = \frac{e^y}{t}$ $\Rightarrow$ $\frac{dy}{dt} = \frac{e^y}{t}$

$$\Rightarrow \frac{1}{e^y} dy = \frac{1}{t} dt \Rightarrow \int e^{-y} dy = \int \frac{1}{t} dt$$

$$\Rightarrow -e^{-y} = \ln|t| + c$$

$$\Rightarrow e^{-y} = -\ln|t| + c$$

---

d. $y'(t) = y(4t^3 + 1), \quad y(0) = 2$

$$\frac{dy}{dt} = y(4t^3+1) \Rightarrow \frac{1}{y} dy = (4t^3+1) dt$$

$$\Rightarrow \int \frac{1}{y} dy = \int (4t^3+1) dt \Rightarrow \ln|y| = t^4 + t + c$$

$$\boxed{y = e^{t^4+t+c} = Ce^{t^4+t}}$$