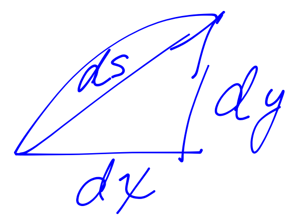
2 arc length

$$y = \frac{1}{3}(x^2+2)^{3/2} \qquad 0 \le x \le 1$$

$$L = \int ds \qquad ds = \sqrt{dx^2 + dy^2}$$



$$= \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{1}{3}\cdot\frac{3}{2}(x^2+2)^{1/2}\cdot 2x = x(x^2+2)^{1/2}$$

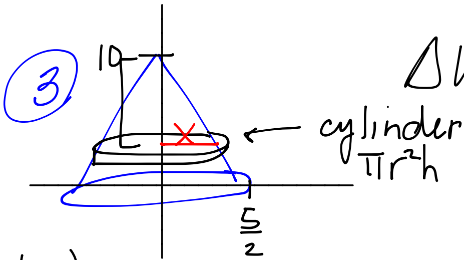
$$\left(\frac{dy}{dx}\right)^2 = x^2(x^2+2) = x^4+2x^2$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+2x^2+x^4 = (1+x^2)^2$$

$$L = \int_0^1 \sqrt{(1+x^2)^2}\,dx$$

$$= \int_0^1 (1+x^2)\,dx = \left(x+\frac{1}{3}x^3\right)\Big|_0^1 = \boxed{\frac{4}{3}}$$

3

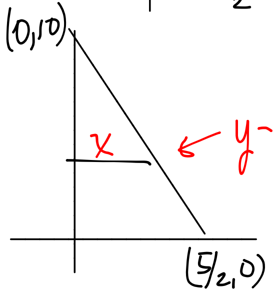


$$\Delta W_s = \underbrace{\text{Force}}\cdot \text{dist}$$

$$= \rho \cdot \text{Volume} \cdot \text{dist}$$

$$= \rho\, \pi r^2\, dy\, (10-y)$$

$$= \rho\, \pi \left(\frac{10-y}{4}\right)^2 (10-y)\, dy$$



$$y - 0 = -4(x - 5/2)$$

$$y = -4x + 10$$

$$x = \frac{y-10}{-4} = \frac{10-y}{4}$$

$$W = \int_0^{10} \frac{\pi\rho}{16} (10-y)^3\, dy$$

$$m = \frac{10-0}{0-5/2} = 10\left(\frac{-2}{5}\right) = -4$$

④

ⓐ $\int x \arctan x \, dx$

$u = \arctan x \qquad dv = x\,dx$

$du = \frac{1}{1+x^2}dx \qquad v = \frac{x^2}{2}$

$\int u\,dv = uv - \int v\,du$

$= \frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}dx$

$= \frac{x^2}{2}\arctan x - \frac{1}{2}\int \left(\cancel{\frac{x^2+1}{1+x^2}}^{\,1} - \frac{1}{1+x^2} \right) dx$

$\frac{x^2}{2}\arctan x - \frac{1}{2}\left[x - \arctan x \right] + C$

ⓑ $\int \frac{x^2+8x-3}{x^3+3x^2}dx \qquad \frac{x^2+8x-3}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$

$x^2+8x-3 = Ax(x+3) + B(x+3) + Cx^2$

$Ax^2 + 3xA \quad + Bx \; + 3B + Cx^2$

$x=0 \quad -3 = A(0) + B(3) + C(0)$

$-3 = 3B \qquad B = -1$

$x=-3 \quad 9-24-3 = A(0) + B(0) + 9C$

$-18 = 9C \qquad C = -2$

$x^2 = Ax^2 + Cx^2 \Rightarrow 1 = A + C$

$1 = A - 2$

$A = 3$

$\int \left(\frac{3}{x} - \underbrace{\frac{1}{x^2}}_{}^{x^{-2}} - \frac{2}{x+3} \right) dx = 3\ln|x| + \frac{1}{x} - 2\ln|x+3| + C$

ⓒ $\displaystyle\int \frac{x^3}{\sqrt{x^2+9}}\,dx$

$x = 3\tan\theta$

x^2+9

$(3\tan\theta)^2+9 = 9\tan^2\theta+9$

$* \; dx = 3\sec^2\theta\, d\theta \qquad = 9(\tan^2\theta+1) = 9\sec^2\theta$

$$= \int \frac{27\tan^3 x}{\sqrt{9\sec^2\theta}}\, 3\sec^2\theta\, d\theta$$

$$= \int \frac{27\tan^3\theta}{3\sec\theta} \cdot 3\sec^2\theta\, d\theta$$

$$= 27\int \underbrace{\tan^3\theta}_{\tan^2\theta\tan\theta}\sec\theta\, d\theta = 27\int (\sec^2\theta-1)\tan\theta\sec\theta\, d\theta$$

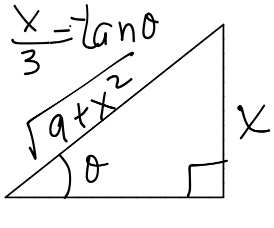
$u = \sec\theta$

$du = \sec\theta\tan\theta$

$$27\int (u^2-1)\,du = 27\left(\frac{u^3}{3} - u\right)$$

$$= 27\left(\frac{\sec^3\theta}{3} - \sec\theta\right) + C$$

$\frac{x}{3} = \tan\theta$



$$= 27\left(\frac{1}{3}\left(\frac{(9+x^2)^{3/2}}{27}\right) - \frac{(9+x^2)^{1/2}}{3}\right) + C$$

$\sec\theta = \frac{\sqrt{9+x^2}}{3}$

(d) $\displaystyle\int \frac{x^2}{(4-x^2)^{3/2}}\,dx = *$

$x = 2\sin\theta$

$4 - x^2 = 4 - 4\sin^2\theta$

$= 4(1-\sin^2\theta)$

$= 4\cos^2\theta$

$*\ dx = 2\cos\theta\, d\theta$

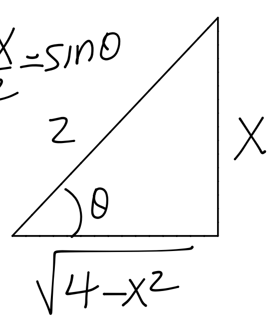
$$* \int \frac{4\sin^2\theta}{(4\cos^2\theta)^{3/2}} \cdot 2\cos\theta\, d\theta$$

$$= \int \frac{4\sin^2\theta}{8\cos^3\theta}\, 2\cos\theta\, d\theta$$

$$= \int \frac{\sin^2\theta}{\cos^2\theta}\, d\theta = \int \tan^2\theta\, d\theta$$

$$= \int (\sec^2\theta - 1)\, d\theta = \tan\theta - \theta + C$$

$\frac{x}{2} = \sin\theta$



$$= \frac{x}{\sqrt{4-x^2}} - \sin^{-1}\left(\frac{x}{2}\right) + C$$

e) $\int_1^3 \frac{1}{\sqrt[3]{x-2}}\,dx = \int_1^2 \frac{1}{\sqrt[3]{x-2}}\,dx + \int_2^3 \frac{1}{\sqrt[3]{x-2}}\,dx = -\frac{3}{2} + \frac{3}{2} = \underline{\underline{0}}$

$$\int_1^2 \frac{1}{(x-2)^{1/3}}\,dx = \lim_{B\to 2^-} \frac{3}{2}(x-2)^{2/3}\Big|_1^B$$

$$= \lim_{B\to 2^-} \left(\frac{3}{2}(\cancel{B-2})^{2/3} - \frac{3}{2}(\cancel{1-2})^{2/3}\right] = -\frac{3}{2}$$

$$\int_2^3 (x-2)^{-1/3}\,dx = \lim_{B\to 2^+}\left[\frac{3}{2} - \frac{3}{2}(\cancel{B-2})^{2/3}\right] = \frac{3}{2}$$

(6) $f(x) = \dfrac{1}{2x-5} \qquad x_0 = 3$

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k$$

k	
0	$f(x) = (2x-5)^{-1}$
1	$-1(2x-5)^{-2}(2)$
2	$1(2)(2x-5)^{-3}\,2^2$
3	$-1\cdot 2\cdot 3(2x-5)^{-4}\,2^3$
4	$1\cdot 2\cdot 3\cdot 4(2x-5)^{-5}\,2^4$

$$f^{(k)}(x) = (-1)^k k!\,(2x-5)^{-(k+1)}\,2^k$$

$$f^{(k)}(3) = (-1)^k k!\,\cancel{1^{-(k+1)}}\,2^k$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \cancel{k!}\,2^k}{\cancel{k!}}(x-3)^k$$

$$= \sum_{k=0}^{\infty} (-1)^k 2^k (x-3)^k$$

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n}$$

$$\lim_{n\to\infty}\left|a_{n+1}\cdot\frac{1}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x+2)^{n+1}}{(n+1)4^{n+1}}\cdot\frac{n4^n}{(x+2)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{\cancel{(x+2)^n}(x+2)}{(n+1)\cancel{4^n}\cdot 4}\cdot\frac{n\cancel{4^n}}{\cancel{(x+2)^n}}\right| = \frac{|x+2|}{4}\lim_{n\to\infty}\cancel{\frac{n}{n+1}} = \frac{|x+2|}{4} < 1$$

$$\Rightarrow -1 < \frac{x+2}{4} < 1 \Rightarrow \underset{-2}{-4} < \underset{-2}{x+2} < \underset{-2}{4} \Rightarrow \underline{\underline{-6 < x < 2}}$$

$$\boxed{R=4}$$

$-6 = x \Rightarrow \sum \frac{(-4)^n}{n4^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ alt harmonic series converges

$x = 2 \quad \sum \frac{4^n}{n4^n} = \sum \frac{1}{n}$ harmonic series $(p=1)$ diverges

IOC $[-6, 2)$

⑧ $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/4}}$

Abs conv $\displaystyle\sum |a_n| = \sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$ p-series $p = 1/4 < 1$

$\therefore$ it diverges

(It does not conv abs)

Alt Series Test

① $\displaystyle\lim_{n\to\infty} \frac{1}{n^{1/4}} = 0$

② Show a_n dec $\quad a'(x) = -\frac{1}{4} x^{-5/4} = \frac{-1}{4x^{5/4}} < 0 \;\; \forall x \geq 1 \quad a_n$ is dec.

$\therefore$ By AST $\displaystyle\sum \frac{(-1)^{n+1}}{n^{1/4}}$ conv.

$\sum |a_n|$ div but $\sum a_n$ conv $\Rightarrow$ cond conv

⑨ $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$

looks like $\displaystyle\sum \frac{n^2}{n^4} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ p-series

$P = 2 > 1$ Convergent.

$$\lim_{n\to\infty} \frac{\frac{n^2-1}{3n^4+1}}{1/n^2} = \lim \frac{n^2(n^2-1)}{3n^4+1}$$

$$= \lim_{n\to\infty} \frac{n^4-n^2}{3n^4+1} = \frac{1}{3} \quad 0 < L < \infty$$

$\therefore$ By LCT $\displaystyle\sum \frac{n^2-1}{3n^4+1}$ also conv.

10 $\sqrt[6]{-64i}$ $\qquad z_k^6 = -64i$

$-64i = \underset{R}{64}\left(\cos\left(\underbrace{\frac{3\pi}{2}+2k\pi}_{\theta}\right)+i\sin\left(\frac{3\pi}{2}+2k\pi\right)\right)$

$z_k = r(\cos\phi + i\sin\phi)$

$r^6 = R \qquad r = \sqrt[6]{R} = \sqrt[6]{64} = 2$

$6\phi = \theta \Rightarrow 6\phi = \frac{3\pi}{2} + 2k\pi$

$\phi = \frac{3\pi}{12} + \frac{2k\pi}{6} = \frac{3\pi}{12} + \frac{4\pi k}{12}$

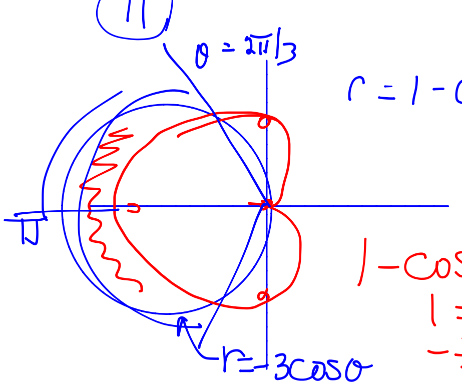
$\frac{3\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{15\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$z_1 = 2e^{i3\pi/12} = 2\left(\cos\frac{3\pi}{12} + i\sin\frac{\pi}{4}\right) = 2cis\left(\frac{3\pi}{12}\right)$

$z_2 = 2e^{i7\pi/12}$

$z_3 = 2e^{i11\pi/12}$

11



$r = 1 - \cos\theta$

$1 - \cos\theta = -3\cos\theta$

$1 = -2\cos\theta$

$-\frac{1}{2} = \cos\theta$

$\theta = 2\pi/3, 4\pi/3$

$$2\int_{2\pi/3}^{\pi} \frac{1}{2}\left[(-3\cos\theta)^2 - (1-\cos\theta)^2\right] d\theta$$

$$= \int_{2\pi/3}^{\pi} \left(9\cos^2\theta - (1 - 2\cos\theta + \cos^2\theta)\right) d\theta$$

$$= \int_{2\pi/3}^{\pi} 8\cos^2\theta \, d\theta - \int_{2\pi/3}^{\pi} (1 - 2\cos\theta) \, d\theta$$

$$= 8\cdot\frac{1}{2}\int_{2\pi/3}^{\pi}(1+\cos 2\theta)\, d\theta - \left[\theta - 2\sin\theta\right]_{2\pi/3}^{\pi}$$

(13) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-\frac{1}{t^2}\cdot 2t}{e^{\sqrt{t}}\cdot\frac{1}{2\sqrt{t}}}$

$$m=\left.\frac{dy}{dx}\right|_{t=1} = \frac{1-\frac{1}{1}\cdot 2}{e^1\frac{1}{2}} = \frac{-1}{\frac{e}{2}} = \frac{-2}{e}$$

$x(1)=e^{\sqrt{1}}=e$

$y(1)=1-\ln 1^2=1$

$$y-y_0=m(x-x_0)$$

$$\boxed{y-1=\frac{-2}{e}(x-e)}$$

(14) $x=4\sqrt{t}$ $\quad y=\frac{t^3}{3}+\frac{1}{2t^2}$ $\quad 1\le t\le 4$

$$ds=\sqrt{dx^2+dy^2}=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$\frac{dx}{dt}=4\cdot\frac{1}{2\sqrt{t}}=\frac{2}{t^{1/2}}$$

$$\frac{dy}{dt}=\frac{3t^2}{3}+\frac{1}{2}\cdot -2t^{-3}=t^2-\frac{1}{t^3}$$

$$\int_1^4\left(t^2+\frac{1}{t^3}\right)dt$$

= Solve

$$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=\frac{4}{t}+t^4-\frac{2}{t}+\frac{1}{t^6}$$

$$=t^4+\frac{2}{t}+\frac{1}{t^6}=\left(t^2+\frac{1}{t^3}\right)^2$$

polar $\int \sqrt{(r')^2 + r^2}\, d\theta$

$r = 1 + \cos\theta$

$x = r\cos\theta \qquad y = r\sin\theta$

$X = (1+\cos\theta)\cos\theta \qquad y = (1+\cos\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-\sin\theta\sin\theta + (1+\cos\theta)\cos\theta}{-\sin\theta\cos\theta - (1+\cos\theta)\sin\theta}$$

15

$$\frac{3+2i}{1+i}\cdot\frac{1-i}{1-i}=\frac{3+2-3i+2i}{1+1}=\boxed{\frac{5}{2}-\frac{i}{2}}$$

b

$$\left(\frac{1}{2}+\frac{1}{2}i\right)^{15}=\left(\frac{1}{\sqrt{2}}e^{i\pi/4}\right)^{15}=\frac{1}{2^{15/2}}e^{i15\pi/4}$$

quad I

$$z^n=(re^{i\theta})^n=r^ne^{in\theta}$$

$$=r^n\operatorname{cis} n\theta$$

$$\sqrt{\frac{1}{4}+\frac{1}{4}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

$$\theta=\arctan\left(\frac{\frac{1}{2}}{1/2}\right)=\pi/4$$

$$=\frac{1}{2^{15/2}}\left(\cos\cancel{\frac{15\pi}{4}}^{7\pi/4}+i\sin\cancel{\frac{15\pi}{4}}^{7\pi/4}\right)$$

$$=\frac{1}{2^{15/2}}\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)$$

$$=\frac{1}{2^8}(1-i)$$

c

$$|-1+2\sqrt{2}i|=\sqrt{z\bar{z}}=\sqrt{x^2+y^2}=r$$

$$=\sqrt{1+8}=\sqrt{9}=\boxed{3}$$