Exam 4 **Name** Solution

Math 132-01 **Calculus II** **November 30, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 65 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.

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1. (8 points) To be completed once exams are graded and returned. Please correct any problem with points deducted. All corrections should be completed neatly on a separate sheet of paper. Once you have finished your corrections, take your exam and corrections to the Office of Student Learning (OSL), and a tutor will check your answers and sign below. The checked solutions should be given to your instructor.

   Signature: ______________________________
   Print Name: ______________________________
   Date: ________________

Q2

2. For the point with rectangular coordinates $(-1, \sqrt{3})$, find polar coordinates with

a. (3 points) $r > 0$ and $0 \le \theta < 2\pi$;

$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3}$

$= \sqrt{4} = 2$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} \Rightarrow \theta = \frac{2\pi}{3}$

$\boxed{(r, \theta) = (2, 2\pi/3)}$

b. (3 points) $r < 0$ and $0 \le \theta < 2\pi$.

$\theta = \frac{5\pi}{3} \Rightarrow \boxed{(r, \theta) = (-2, 5\pi/3)}$

3. (8 points) Convert the polar equation $r = 2\cos\theta - 6\sin\theta$ to rectangular coordinates and describe the curve it represents.

$r = 2\cos\theta - 6\sin\theta \Rightarrow r^2 = 2r\cos\theta - 6r\sin\theta$

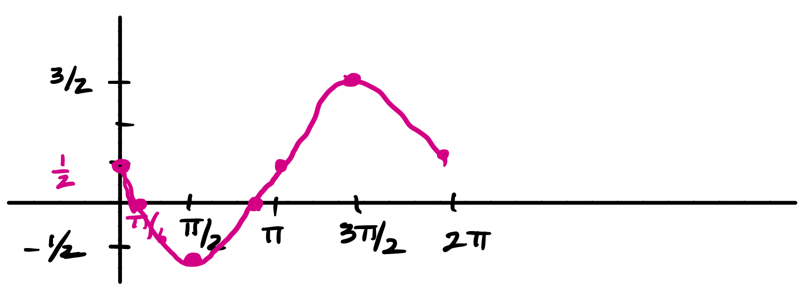
$\Rightarrow x^2 + y^2 = 2x - 6y \Rightarrow x^2 - 2x + 1 - 1 + y^2 + 6y + 9 - 9 = 0$

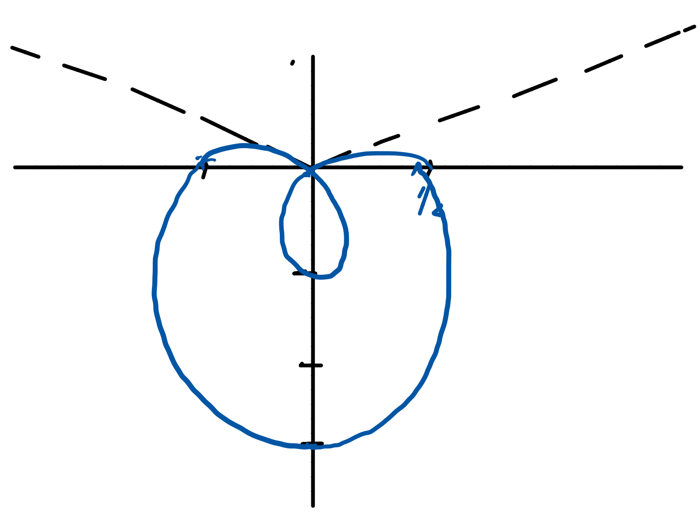
$\Rightarrow$ $\boxed{(x-1)^2 + (y+3)^2 = 10 \quad \text{Circle centered at } (1,-3) \text{ w/ rad } \sqrt{10}}$

4. (8 points) Sketch the polar curve $r = \frac{1}{2} - \sin\theta$.

$\frac{1}{2} - \sin\theta = 0$

$\frac{1}{2} = \sin\theta \qquad \theta = \pi/6,\ 5\pi/6$





5. (8 points) Set up, but do not evaluate, an integral representing the area enclosed by the inner loop of $r = \frac{1}{2} - \sin\theta$.

$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left(\frac{1}{2} - \sin\theta\right)^2 d\theta$$

6. Consider the parametric equations $x = e^t$, $y = 3e^{-2t}$ for $0 \leq t \leq \ln 3$.

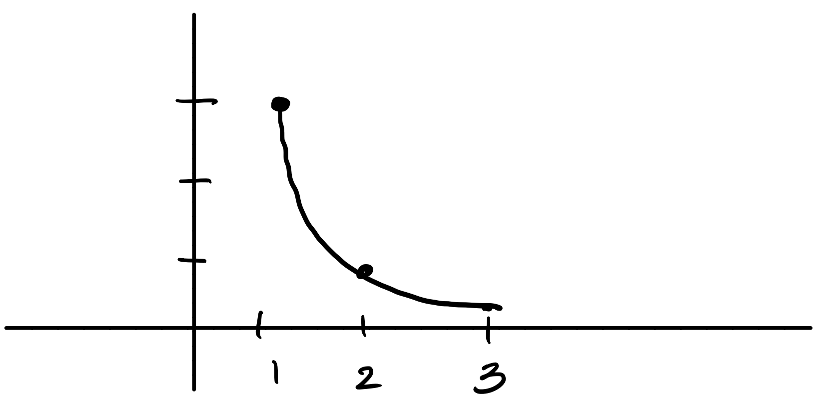
a. (6 points) Eliminate the parameter to obtain an equation in x and y.

$x = e^t$

$0 \leq t \leq \ln 3$

$$\Rightarrow \boxed{y = 3(e^t)^{-2} = 3x^{-2} = \frac{3}{x^2} \qquad 1 \leq x \leq 3}$$

b. (4 points) Sketch the curve.



7. Write each complex number in the form $x + yi$.

a. (4 points) $i^{77} = (i^4)^{19} \cdot i$

$= \boxed{i}$

$$\begin{array}{r} 19 \\ 4\overline{)77} \\ \underline{4} \\ 37 \\ \underline{36} \\ 1 \end{array}$$

b. (4 points) $\frac{2-3i}{-1+5i} \cdot \frac{-1-5i}{-1-5i} = \frac{-2-15+3i-10i}{1+25}$

$$= \boxed{\frac{-17}{26} - \frac{7}{26}i}$$

8. (8 points) Find the point (x, y) where the tangent line to the parametric curve $x = t^2 - 9$, $y = t^2 - 8t$ is horizontal.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t-8}{2t} = 0$$

if $2t - 8 = 0 \quad t = 4$

$\Rightarrow$ pt $(16-9, \; 16-32) = \boxed{(7, -16)}$

9. (8 points) Find an equation of the tangent line to the polar curve $r = 8\sin\theta$ at $\theta = \frac{5\pi}{6}$.

$x = r\cos\theta \Big|_{\theta = 5\pi/6} = 4\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$

$y = r\sin\theta \Big|_{\theta=5\pi/6} = 4\left(\frac{1}{2}\right) = 2$

$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}\Big|_{\theta = 5\pi/6}$

$= \frac{-4\sqrt{3}\left(\frac{1}{2}\right) + 4\left(\sqrt{3}/2\right)}{-4\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) - 4\left(\frac{1}{2}\right)} = \frac{-2\sqrt{3} + 2\sqrt{3}}{6 - 2} = 0$

$r = 8\sin 5\pi/6$
$= 8\left(\frac{1}{2}\right) = 4$

$r' = 8\cos\theta$

$r'\big|_{\theta = \frac{5\pi}{6}} = 8\left(-\frac{\sqrt{3}}{2}\right)$
$= -4\sqrt{3}$

$y - 2 = 0(x + 2\sqrt{3})$

$\Rightarrow \boxed{y = 2}$

10. (8 points) Write $(-1 + i)^8$ in the form $x + yi$ without expanding.

$-1 + i \Rightarrow r = \sqrt{1+1} = \sqrt{2} \qquad \tan\theta = \frac{1}{-1} \Rightarrow \theta = \frac{3\pi}{4}$

$-1 + i = \sqrt{2}\, e^{i 3\pi/4}$

$(-1+i)^8 = \left(\sqrt{2}\, e^{i3\pi/4}\right)^8 = 2^4 e^{i 6\pi}$

$= 16(\cos 6\pi + i \sin 6\pi)$

$= \boxed{16}$

11. (8 points) Find all complex cube roots of -27.

$-27 = 27(\cos\pi + i\sin\pi) = 27e^{i\pi}$

$r = \sqrt[3]{27} = 3$

$3\theta = \pi + 2k\pi$

$\theta = \frac{\pi}{3} + \frac{2k\pi}{3}$

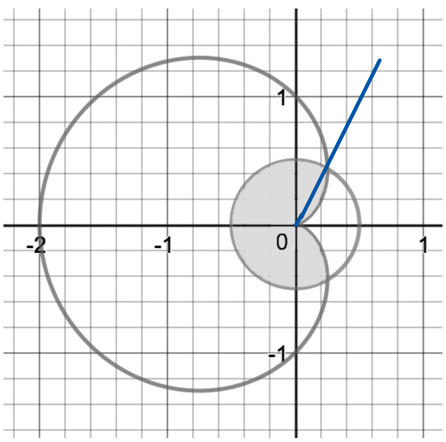
$\pi/3, \quad \pi, \quad 5\pi/3$

$w_1 = 3e^{i\pi/3} = 3(\cos\pi/3 + i\sin\pi/3)$
$= 3\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$w_2 = 3e^{i\pi} = -3$

$w_3 = 3(\cos 5\pi/3 + i\sin 5\pi/3)$
$= 3\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

12. (12 points) Calculate the area inside both the cardioid $r = 1 - \cos\theta$ and the circle $r = \frac{1}{2}$.



$1 - \cos\theta = \frac{1}{2}$

$\frac{1}{2} = \cos\theta \qquad \theta = \pi/3$

$A = 2\cdot\frac{1}{2}\int_0^{\pi/3}(1-\cos\theta)^2\,d\theta + 2\cdot\frac{1}{2}\int_{\pi/3}^{\pi}\left(\frac{1}{2}\right)^2 d\theta$

$A = \int_0^{\pi/3}\left(1 - 2\cos\theta + \overset{\frac{1}{2}(1+\cos 2\theta)}{\cos^2\theta}\right)d\theta + \frac{1}{4}\theta\Big|_{\pi/3}^{\pi}$

$= \theta\Big|_0^{\pi/3} - 2\sin\theta\Big|_0^{\pi/3} + \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi/3} + \frac{1}{4}\theta\Big|_{\pi/3}^{\pi}$

$= \frac{\pi}{3} - 2\sin\frac{\pi}{3} + \frac{1}{2}\left(\frac{\pi}{3}\right) + \frac{1}{4}\sin\frac{2\pi}{3} + \frac{1}{4}(\pi - \pi/3)$

$= \frac{\pi}{3} - 2\frac{\sqrt{3}}{2} + \frac{\pi}{6} + \frac{1}{4}\frac{\sqrt{3}}{2} + \frac{1}{4}\frac{2\pi}{3}$

$= \frac{\pi}{3} + \frac{\pi}{6} + \frac{\pi}{6} - \sqrt{3} + \frac{\sqrt{3}}{6} = \boxed{\frac{2\pi}{3} - \frac{5\sqrt{3}}{6}}$

Question	Points	Score
1	8	
2	6	
3	8	
4	8	
5	8	
6	10	
7	8	
8	8	
9	8	
10	8	
11	8	
12	12	
Total:	100	