

Guidelines

- **Calculators are not allowed.**
 - Read the questions carefully. You have 65 minutes; use your time wisely.
 - You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
 - Put a box around your final answers when relevant.
 - Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
 - Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.
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1. (8 points) To be completed once exams are graded and returned. Please correct any problem with points deducted. All corrections should be completed neatly on a separate sheet of paper. Once you have finished your corrections, take your exam and corrections to the Office of Student Learning (OSL), and a tutor will check your answers and sign below. The checked solutions should be given to your instructor.

Signature: _____

Print Name: _____

Date: _____

Tests	Conditions	Conclusion
Positive Term Tests		
Integral Test $a_k = a(k)$, $a(x)$ is positive, continuous, decreasing	$\int_1^{\infty} a(x)dx$ converges	Converges
	$\int_1^{\infty} a(x)dx$ diverges	Diverges
Comparison Test	$0 \leq a_k \leq b_k$ and $\sum_{k=1}^{\infty} b_k$ converges	Converges
	$0 \leq b_k \leq a_k$ and $\sum_{k=1}^{\infty} b_k$ diverges	Diverges
Limit Comparison Test $\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = L$, $0 < L < \infty$	$\sum_{k=1}^{\infty} b_k$ converges	Converges
	$\sum_{k=1}^{\infty} b_k$ diverges	Diverges
Ratio Test $\lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = \rho$	$\rho < 1$	Converges
	$\rho > 1$	Diverges
	$\rho = 1$	Inconclusive
Root Test $\lim_{k \rightarrow \infty} [a_k]^{1/k} = \rho$	$\rho < 1$	Converges
	$\rho > 1$	Diverges
	$\rho = 1$	Inconclusive
Series with some non-positive terms		
Alternating Series Test $a_k > 0$ and $0 < a_{k+1} \leq a_k$	$\lim_{k \rightarrow \infty} a_k = 0$	Converges
	$\lim_{k \rightarrow \infty} a_k \neq 0$	Use Divergence Test to show divergent

Question	Points	Score
1	8	
2	6	
3	6	
4	6	
5	6	
6	8	
7	6	
8	10	
9	8	
10	8	
11	8	
12	8	
13	12	
Total:	100	

2. (6 points) Determine whether the sequence $\left\{ \frac{k^2 + 7}{\sqrt{9k^4 + 1}} \right\}$ is convergent or divergent.

3. (6 points) Determine if the series $\sum_{k=1}^{\infty} k^{-4/5}$ is convergent or divergent. Explain your reasoning.

4. (6 points) Determine if the series $\sum_{k=1}^{\infty} \frac{k^3}{k^3 + 1}$ is convergent or divergent. Explain your reasoning.

5. (6 points) Determine if the series $\frac{1}{16} + \frac{3}{64} + \frac{9}{256} + \frac{27}{1024} + \dots$ is convergent or divergent. Explain your reasoning.

6. (8 points) Use the integral test to determine if the series $\sum_{k=2}^{\infty} \frac{1}{k\sqrt[3]{\ln k}}$ is convergent or divergent (be sure to show that three conditions apply).

7. (6 points) Find the sum of the series $\sum_{k=3}^{\infty} \frac{2}{3^k}$.

8. (10 points) Determine if the series $\sum_{k=1}^{\infty} \left(\frac{1}{k+1} - \frac{1}{k+3} \right)$ is convergent or divergent. If it converges, what is the sum?

9. (8 points) Determine if the series $\sum_{k=1}^{\infty} \frac{\cos k}{k^3}$ converges or diverges. Explain your reasoning.

10. (8 points) Determine if the series $\sum_{k=1}^{\infty} \frac{2k+1}{4^k}$ converges or diverges. Explain your reasoning.

11. (8 points) Determine if the series $\sum_{k=1}^{\infty} \left(\frac{2k^2}{3k^2+1} \right)^k$ converges or diverges. Explain your reasoning.

12. (8 points) Determine if the series $\sum_{k=1}^{\infty} \frac{k^2}{k^4 + k^2 + 2}$ converges or diverges. Explain your reasoning.

13. Given the series $\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{k^3 + 32}$,

a. (6 points) Show that the series converges. Show all your work, explain your answer.

b. (6 points) Determine whether the series converges absolutely or conditionally.