

Guidelines

- **Calculators are not allowed.**
 - Read the questions carefully. You have 65 minutes; use your time wisely.
 - You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
 - Put a box around your final answers when relevant.
 - Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
 - Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.
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1. (8 points) To be completed once exams are graded and returned. Please correct any problem with points deducted. All corrections should be completed neatly on a separate sheet of paper. Once you have finished your corrections, take your exam and corrections to the Office of Student Learning (OSL), and a tutor will check your answers and sign below. The checked solutions should be given to your instructor.

Signature: _____

Print Name: _____

Date: _____

Tests	Conditions	Conclusion
Positive Term Tests		
Integral Test $a_k = a(k)$, $a(x)$ is positive, continuous, decreasing	$\int_1^{\infty} a(x)dx$ converges	Converges
	$\int_1^{\infty} a(x)dx$ diverges	Diverges
Comparison Test	$0 \leq a_k \leq b_k$ and $\sum_{k=1}^{\infty} b_k$ converges	Converges
	$0 \leq b_k \leq a_k$ and $\sum_{k=1}^{\infty} b_k$ diverges	Diverges
Limit Comparison Test $\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = L$, $0 < L < \infty$	$\sum_{k=1}^{\infty} b_k$ converges	Converges
	$\sum_{k=1}^{\infty} b_k$ diverges	Diverges
Ratio Test $\lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = \rho$	$\rho < 1$	Converges
	$\rho > 1$	Diverges
	$\rho = 1$	Inconclusive
Root Test $\lim_{k \rightarrow \infty} [a_k]^{1/k} = \rho$	$\rho < 1$	Converges
	$\rho > 1$	Diverges
	$\rho = 1$	Inconclusive
Series with some non-positive terms		
Alternating Series Test $a_k > 0$ and $0 < a_{k+1} \leq a_k$	$\lim_{k \rightarrow \infty} a_k = 0$	Converges
	$\lim_{k \rightarrow \infty} a_k \neq 0$	Use Divergence Test to show divergent

Question	Points	Score
1	8	
2	12	
3	8	
4	8	
5	8	
6	8	
7	8	
8	16	
9	12	
10	12	
Total:	100	

2. Determine whether the following sequences converge or diverge. If it is convergent, find its limit.

a. (6 points) $\left\{ \frac{3 + 5k^2}{k + k^2} \right\}$

b. (6 points) $\left\{ \frac{(2n - 1)!}{(2n + 1)!} \right\}$

3. (8 points) Find the sum of the series $\sum_{k=0}^{\infty} \frac{3^k}{\pi^{k+1}}$.

4. (8 points) Determine if the series $\sum_{k=1}^{\infty} \frac{k^{2k}}{(1 + 2k^2)^k}$ converges or diverges. Explain your reasoning.

5. (8 points) Determine if the series $\sum_{k=1}^{\infty} \frac{\sin(4k)}{4^k}$ converges or diverges. Explain your reasoning.

6. (8 points) Determine if the series $\sum_{k=1}^{\infty} \frac{(k+1)^2}{k(k+2)}$ converges or diverges. Explain your reasoning.

7. (8 points) Determine if the series $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ converges or diverges. Explain your reasoning.

8. a. (8 points) Show the series $\sum_{k=1}^{\infty} \frac{(-1)^k k}{k^2 + 100}$ converges. Explain your reasoning.

b. (8 points) Determine if the series $\sum_{k=1}^{\infty} \frac{(-1)^k k}{k^2 + 100}$ is absolutely convergent or conditionally convergent. Explain your reasoning.

9. (12 points) Find the Taylor Series for $f(x) = \frac{1}{2x + 2}$ at $a = 1$.

10. (12 points) Determine the radius and interval of convergence for the series

$$\sum_{k=1}^{\infty} \frac{x^k}{k 3^k}$$