Exam 2 Version A **Name** Solution

Math 132-01 **Calculus II** **February 28, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 65 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- **Put a box around your final answers when relevant.**
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.

Question	Points	Score
1	8	
2	8	
3	8	
4	10	
5	10	
6	12	
7	12	
8	10	
9	10	
10	12	
Total:	100	

1. (8 points) On a separate sheet of paper, correct any problem where points were deducted. Take the test corrections and the exam to the OSL and have a tutor check and sign your corrections. Return your test corrections to your instructor.

2. (8 points) Evaluate $\int \frac{x^2 - 11}{x + 3}$

$$\int \frac{x^2-11}{x+3}\,dx = \int \left(x-3 - \frac{2}{x+2}\right)dx$$

$$\boxed{= \frac{1}{2}x^2 - 3x - 2\ln|x+2| + C}$$

$$\begin{array}{r} x - 3 \\ x+3 \overline{)\, x^2 + 0x - 11} \\ -(x^2 + 3x) \\ \hline -3x - 11 \\ -(-3x - 9) \\ \hline -2 \end{array}$$

---

3. (8 points) Evaluate $\int \frac{1}{x^2 + 2x + 5}\,dx$

$$\int \frac{1}{x^2+2x+5}\,dx = \int \frac{1}{(x+1)^2+4}\,dx = *$$

Complete the square

$x^2 + 2x + 1 - 1 + 5$

$= (x+1)^2 + 4$

$$\boxed{= \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + C}$$

might see

$$* = \int \frac{1}{u^2+4}\,du = \frac{1}{4}\int \frac{1}{\left(\frac{u}{2}\right)^2+1}\,du = \frac{2}{4}\int \frac{1}{w^2+1}\,dw$$

these steps are not required but partial credit should be used if some/all are present.

4. (10 points) Evaluate $\int \arctan x\,dx$.

$u = \arctan x \qquad dv = dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = x$

$$\int \arctan x$$

$$= x\arctan x - \int \frac{x}{1+x^2}\,dx$$

$$= x\arctan x - \frac{1}{2}\ln|1+x^2|\,dx$$

might see: $\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$ $\qquad u = 1+x^2$, $du = 2x\,dx$

$$= \frac{1}{2}\ln|u| + C$$

5. (10 points) Evaluate $\int \frac{\sin^3 x}{\cos^6 x}\,dx.$ $= \int \tan^3 x \sec^3 x \, dx$

$$= \int \tan^2 x \cdot \sec^2 x \cdot \sec x \tan x \, dx$$

$$= \int (\sec^2 x - 1)\sec^2 x \cdot \sec x \tan x \, dx$$

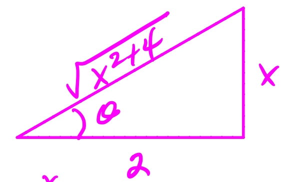
$u = \sec x$
$du = \sec x \tan x \, dx$

$$= \int (u^2 - 1) u^2 \, du = \int (u^4 - u^2)\, du$$

$$\boxed{= \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}$$

6. (12 points) Evaluate $\int \frac{x^4}{(x^2+4)^{9/2}}\,dx$

$x^2 + 4 = 4\tan^2\theta + 4 = 4\sec^2\theta$
$x = 2\tan\theta$
$dx = 2\sec^2\theta \, d\theta$



$\tan\theta = \frac{x}{2}$ $\quad \sin\theta = \frac{x}{\sqrt{x^2+4}}$

$$\int \frac{x^4}{(x^2+4)^{9/2}}\,dx = \int \frac{2^4 \tan^4\theta \cdot 2\sec^2\theta}{2^9 \sec^9\theta}\,d\theta$$

$$= \frac{1}{2^4}\int \frac{\tan^4\theta}{\sec^7\theta}\,d\theta = \frac{1}{16}\int \sin^4\theta \cos^3\theta \, d\theta$$

$$= \frac{1}{16}\int \sin^4\theta (1 - \sin^2\theta)\cos\theta \, d\theta$$

$u = \sin\theta$
$du = \cos\theta \, d\theta$

$$= \frac{1}{16}\int u^4(1-u^2)\,du = \frac{1}{16}\int (u^4 - u^6)\,du$$

$$= \frac{1}{16}\cdot\left(\frac{1}{5}\sin^5\theta - \frac{1}{7}\sin^7\theta\right) + C$$

$$\boxed{= \frac{1}{16}\left(\frac{1}{5}\frac{x^5}{(x^2+4)^{5/2}} - \frac{1}{7}\frac{x^7}{(x^2+4)^{7/2}}\right) + C}$$

$$= \frac{1}{80}\frac{x^5}{(x^2+4)^{5/2}} - \frac{1}{112}\frac{x^7}{(x^2+4)^{7/2}} + C$$

7. (12 points) Evaluate $\int x^2 \cosh x \, dx$.

$u = x^2$ $\quad dv = \cosh x \, dx$

$du = 2x\,dx$ $\quad v = \sinh x$

$$\int x^2 \cosh x \, dx = x^2 \sinh x - 2\int x \sinh x \, dx = *$$

$u = x$ $\quad dv = \sinh x \, dx$

$du = dx$ $\quad v = \cosh x$

$$* = x^2 \sinh x - 2\left[x\cosh x - \int \cosh x \, dx\right]$$

$$\boxed{= x^2 \sinh x - 2x\cosh x + 2\sinh x + C}$$

8. (10 points) Find the general solution of $x^2 \frac{dw}{dx} = \sqrt{w}(3x+1)$

$$x^2 \frac{dw}{dx} = \sqrt{w}(3x+1)$$

$$\frac{1}{\sqrt{w}} \frac{dw}{dx} = \frac{3x+1}{x^2}$$

$$w^{-1/2} dw = \left(\frac{3}{x} + \frac{1}{x^2}\right) dx$$

$$\int w^{-1/2} dw = \int \left(\frac{3}{x} + \frac{1}{x^2}\right) dx$$

$$\Rightarrow 2\sqrt{w} = 3\ln|x| - \frac{1}{x} + C$$

So

$$\sqrt{w} = \frac{3}{2}\ln|x| - \frac{1}{2x} + C$$

$$\boxed{w = \left(\frac{3}{2}\ln|x| - \frac{1}{2x} + C\right)^2}$$

9. (10 points) Evaluate the following integral, if it exists.

$$\int_1^\infty \frac{2x+3}{(2x^2+6x)^2}$$

$$\int_1^\infty \frac{2x+3}{(2x^2+6x)^2}dx = \lim_{b\to\infty} \int_1^b \frac{2x+3}{(2x^2+6x)^2}dx$$

$$= \lim_{b\to\infty} \left[-\frac{1}{2}\,\frac{1}{2x^2+6x}\right]_1^b = \lim_{b\to\infty}\left[\frac{-1}{2(2b^2+6b)} + \frac{1}{16}\right] = \boxed{\frac{1}{16}}$$

Aside $u = 2x^2+6x$

$du = (4x+6)dx$

$= 2(2x+3)dx$

$$\int \frac{2x+3}{(2x^2+6x)^2}dx = \frac{1}{2}\int \frac{1}{u^2}du$$

$$= -\frac{1}{2}\,\frac{1}{2x^2+6x}$$

10. (12 points) Evaluate $\int \frac{8-x}{x^3+4x}\,dx$

$$\frac{8-x}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$\Rightarrow -x+8 = A(x^2+4) + (Bx+C)x$$

$$\Rightarrow 0x^2 - x + 8 = (A+B)x^2 + Cx + 4A$$

$0x^2 = (A+B)x^2$ $\quad -x = Cx$ $\quad 8 = 4A$

$0 = 2 + B$ $\quad C = -1$ $\quad A = 2$

$B = -2$

$$\int \frac{8-x}{x^3+4x}dx = \int \frac{2}{x}dx - \int \frac{2x}{x^2+4}dx - \int \frac{1}{x^2+4}dx$$

$$\boxed{= 2\ln|x| - \ln|x^2+4| - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C}$$