Exam 2 **Name**____________________________________

Math 132-01 **Calculus II** **October 8, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 65 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.

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1. (8 points) To be completed once exams are graded and returned. Please correct any problem with points deducted. All corrections should be completed neatly on a separate sheet of paper. Once you have finished your corrections, take your exam and corrections to the Office of Student Learning (OSL), and a tutor will check your answers and sign below. The checked solutions should be given to your instructor.

   Signature: ________________________________
   Print Name: ________________________________
   Date: ________________

Question	Points	Score
1	8	
2	8	
3	10	
4	8	
5	10	
6	10	
7	12	
8	12	
9	12	
10	10	
Total:	100	

2. (8 points) Evaluate $\displaystyle\int_{\pi/4}^{\pi/3} \frac{1}{\cos^2 x \tan x}\, dx$

Solution:

$$\int_{\pi/4}^{\pi/3} \frac{1}{\cos^2 x \tan x}\, dx = \int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{\tan x}\, dx$$

Let $u = \tan x \implies u_{upper} = \sqrt{3},\ u_{lower} = 1$

$$\int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{\tan x}\, dx = \int_1^{\sqrt{3}} \frac{1}{u}\, du = \ln|u|\Big|_1^{\sqrt{3}} = \ln\sqrt{3} - \ln 1 = \ln\sqrt{3}$$

3. (10 points) Evaluate $\displaystyle\int 4x \sec^2(2x)\, dx$.

Solution:

$$\begin{array}{ll} u = 4x & dv = \sec^2(2x)\, dx \\ du = 4dx & v = \frac{1}{2}\tan 2x \end{array}$$

$$\begin{aligned} \int 4x\sec^2(2x)\, dx &= 2x\tan 2x - 2\int \tan 2x\, dx \\ &= 2x\tan 2x + \ln|\cos(2x)| + C \end{aligned}$$

4. (8 points) Find the general solution of $\sqrt{x}\,\dfrac{dy}{dx} = e^{y+\sqrt{x}}$

Solution:

$$\sqrt{x}\,\frac{dy}{dx} = e^y e^{\sqrt{x}}$$

$$\int \frac{dy}{e^y} = \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$$

Let $u = \sqrt{x} \implies du = \frac{1}{2\sqrt{x}}\, dx$

$$-e^{-y} = 2\int e^u\, du$$

$$
\begin{aligned}
-e^{-y} &= 2e^{\sqrt{x}} + C \\
e^{-y} &= C - 2e^{\sqrt{x}} \\
-y &= \ln\left(C - 2e^{\sqrt{x}}\right) \\
y &= -\ln\left(C - 2e^{\sqrt{x}}\right)
\end{aligned}
$$

5. (10 points) Evaluate $\displaystyle\int \frac{x^3}{x^2-2x-3}\,dx$.

Solution:

$$
\begin{array}{r|l}
 & x+2 \\ \hline
x^2-2x-3 & x^3 \\
 & -x^3+2x^2+3x \\ \hline
 & 2x^2+3x \\
 & -2x^2+4x+6 \\ \hline
 & 7x+6
\end{array}
$$

So,

$$\int \frac{x^3}{x^2-2x-3}\,dx = \int x+2+\frac{7x+6}{(x-3)(x+1)}\,dx$$

$$\frac{7x+6}{(x-3)(x+1)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$7x+6 = A(x-3) + B(x+1)$$

Let $x=3$.

$$7(3)+6 = A(0)+(4) \implies B = \frac{27}{4}$$

Let $x=-1$

$$-7+6 = A(-1-3)+B(0) \implies A = \frac{1}{4}$$

$$
\begin{aligned}
\int x+2+\frac{7x+6}{(x-3)(x+1)}\,dx &= \int x+2+\frac{1}{4(x+1)}+\frac{27}{4(x-3)}\,dx \\
&= \frac{1}{2}x^2+2x+\frac{1}{4}\ln|x+1|+\frac{27}{4}\ln|x-3|+C
\end{aligned}
$$

6. (10 points) Evaluate $\displaystyle\int \frac{2+\sin x+\cos x}{\cos^2 x}\,dx$.

Solution:

$$\int \frac{2+\sin x+\cos x}{\cos^2 x}\,dx = \int 2\sec^2 x + \frac{\sin x}{\cos^2 x} + \sec x\,dx$$

For the middle term, let $u = \cos x \implies du = -\sin x\,dx$

$$= 2\tan x + \ln|\sec x + \tan x| - \int \frac{1}{u^2}\,du$$

$$= 2\tan x + \ln|\sec x + \tan x| + \frac{1}{u}$$

$$= 2\tan x + \ln|\sec x + \tan x| + \sec x + C$$

7. (12 points) Evaluate $\displaystyle\int x^{3/2}(\ln x)^2\,dx$.

Solution:

$$\begin{array}{ll} u = \ln^2 x & dv = x^{3/2} \\ du = \frac{2\ln x}{x} & v = \frac{2x^{5/2}}{5} \end{array}$$

$$\int x^{3/2}(\ln x)^2\,dx = \frac{2x^{5/2}}{5}\ln^2 x - \frac{4}{5}\int x^{3/2}\ln x\,dx$$

$$\begin{array}{ll} u = \ln x & dv = x^{3/2} \\ du = \frac{1}{x} & v = \frac{2x^{5/2}}{5} \end{array}$$

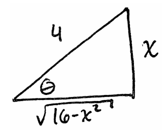
$$= \frac{2x^{5/2}}{5}\ln^2 x - \frac{4}{5}\left(\frac{2x^{5/2}}{5}\ln x - \frac{2}{5}\int x^{3/2}\,dx\right)$$

$$= \frac{2x^{5/2}}{5}\ln^2 x - \frac{8x^{5/2}}{25}\ln x + \frac{16}{125}x^{5/2} + C$$

8. (12 points) Evaluate $\displaystyle\int \frac{x^2}{(16-x^2)^{3/2}}\,dx$

Solution: Draw a triangle, and use trig ratios to determine the following:

$$\begin{aligned} \tfrac{x}{4} &= \sin\theta \\ dx &= 4\cos\theta\,d\theta \\ \tfrac{\sqrt{16-x^2}}{4} &= \cos\theta \end{aligned}$$



$$\begin{aligned} \int \frac{x^2}{(16-x^2)^{3/2}}\,dx &= \int \frac{(4\sin\theta)^2(4\cos\theta)d\theta}{(4\cos\theta)^3} \\ &= \int \tan^2\theta\,d\theta \\ &\int \sec^2\theta - 1\,d\theta \\ &= \tan\theta - \theta + C \\ &= \frac{x}{\sqrt{16-x^2}} - \arcsin\left(\frac{x}{4}\right) + C \end{aligned}$$

9. (12 points) Evaluate $\displaystyle\int \frac{z+1}{z^2(z^2+4)}\,dz$

Solution:

$$\frac{z+1}{z^2(z^2+4)} = \frac{A}{z} + \frac{B}{z^2} + \frac{Cz+D}{z^2+4}$$

$$\begin{aligned} z+1 &= Az(z^2+4) + B(z^2+4) + (Cz+D)z^2 \\ &= Az^3 + 4A + Bz^2 + 4B + Cz^3 + Dz^2 \end{aligned}$$

Equate coefficients:

$$\begin{aligned} z^3\!: &\quad A + C = 0 \\ z^2\!: &\quad B + D = 0 \\ z\!: &\quad 4A = 1 \\ constant\!: &\quad 4B = 1 \end{aligned}$$

From the above:

$$A = B = \tfrac{1}{4} \quad C = D = -\tfrac{1}{4}$$

So,

$$\frac{z+1}{z^2(z^2+4)} = \frac{1}{4z} + \frac{1}{4z^2} - \frac{z}{4(z^2+4)} - \frac{1}{4(z^2+4)}$$

$$\int \frac{z+1}{z^2(z^2+4)} = \frac{1}{4}\int \frac{1}{z} + \frac{1}{z^2} - \frac{z}{z^2+4} - \frac{1}{z^2+4}\,dz$$

$$= \frac{1}{4}\left(\ln|z| - \frac{1}{z} - \frac{1}{2}\ln|z^2+4| - \frac{1}{2}\arctan\left(\frac{z}{2}\right)\right) + C$$

10. (10 points) Evaluate the following integral, if it exists.

$$\int_1^\infty \frac{\ln y}{y^3}\,dy$$

Solution:

$$u = \ln y \quad dv = \tfrac{1}{y^3}dy$$
$$du = \tfrac{1}{y} \quad v = -\tfrac{1}{2y^2}$$

$$\int_1^\infty \frac{\ln y}{y^3}\,dy = \lim_{b\to\infty}\left[\frac{-\ln y}{2y^2}\right]_1^b + \frac{1}{2}\int_1^b \frac{1}{y^3}\,dy$$

$$= \lim_{b\to\infty}\left[\frac{-\ln y}{2y^2} - \frac{1}{4y^2}\right]_1^b$$

$$= \lim_{b\to\infty}\left[\frac{-\ln b}{4b} - \frac{\ln 1}{4(1)^2} - \frac{1}{4b^2} + \frac{1}{4(1)^2}\right]$$

Use L'Hôpital's Rule:

$$= \lim_{b\to\infty}\left(-\frac{1/b}{4b} - \frac{1}{4b^2}\right) + 0 + \frac{1}{4}$$

$$= (0+0) + \frac{1}{4}$$

So the integral converges and has a value of $1/4$