Exam 1 **Name**__________________________________

Math 132-01 **Calculus II** **February 7, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 65 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.

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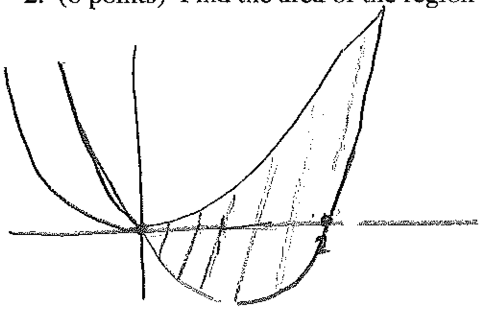
1. (8 points) To be complete once exams are graded and returned. Please correct any problem with points deducted. All corrections should be completed neatly on a separate sheet of paper. Once you have finished your corrections, take your exam and corrections to the Office of Student Learning (OSL), and a tutor will check your answers and sign. The checked solutions should be given to your instructor.

2. (8 points) Find the area of the region bounded by $y = x^2$, $y = 2x^2 - 4x$, and $y = 0$.

$y = x^2$

$y = 2x(x-2)$



$x^2 = 2x^2 - 4x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0, 4$

$A = \int_0^4 x^2 - (2x^2 - 4x)\,dx$

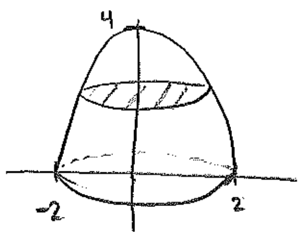
$= \int_0^4 4x - x^2\,dx$

$= \left[2x^2 - \frac{1}{3}x^3\right]_0^4$

$= 32 - \frac{64}{3} = \boxed{\frac{32}{3}}$

3. The region R in the first quadrant bounded by the parabola $y = 4 - x^2$ and the coordinate axes is revolved about the y-axis to produce a dome shaped solid. Find the volume of the solid in the following ways.

   a. (6 points) Apply the disk method. Set up the integral, but do not evaluate.



$V = \int_a^b A(y)\,dy$

$= \int_0^4 \pi r^2\,dy$

$= \int_0^4 \pi x^2\,dy$

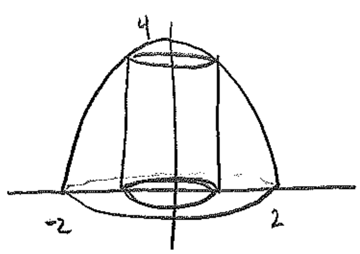
$= \boxed{\int_0^4 \pi (4-y)\,dy}$

$y = 4 - x^2$

$x^2 = 4 - y$

$x = \sqrt{4-y}$

   b. (6 points) Apply the shell method. Set up the integral, but do not evaluate.



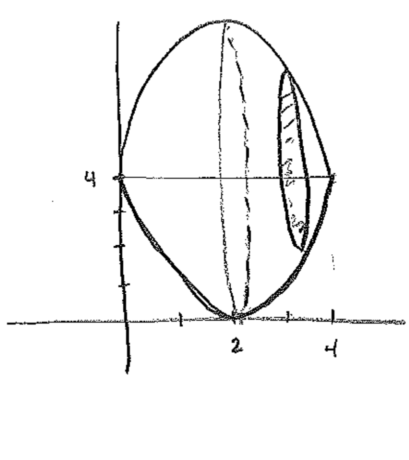
$V = \int_a^b 2\pi\, r(x)\, h(x)\,dx$

$= \int_0^2 2\pi\, r(x)\, h(x)\,dx$

$= \int_0^2 2\pi (x)(y)\,dx$

$= \boxed{\int_0^2 2\pi x(4 - x^2)\,dx}$

4. (6 points) The region bounded by the graphs of $y = (x-2)^2$ and $y = 4$ is revolved about the line $y = 4$. What is the volume of the solid generated? Set up the integral, but do not evaluate.



$V = \int_a^b \pi r^2\,dx$

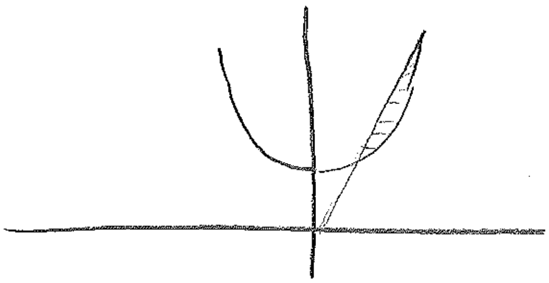
$= \int_0^4 \pi r^2\,dx$

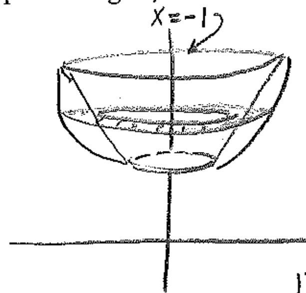
$= \int_0^4 \pi (4-y)^2\,dx$

$= \int_0^4 \pi \left[4 - (x-2)^2\right]^2\,dx$

$= \boxed{\int_0^4 \pi (-x^2 + 4x)^2\,dx}$

5. (6 points) The region bounded by the graphs of $y = 6x$ and $y = x^2 + 5$ is revolved about the line $x = -1$. What is the volume fo the solid generated? Set up the integral, but do not evaluate.





$$V = \int_a^b \pi R^2 - \pi r^2 \, dy$$
$$= \int_1^5 \pi(x_1 - (-1))^2 + \pi(x_2 - (-1))^2 \, dy$$
$$= \int_1^5 \pi\left[\sqrt{y-5}+1\right]^2 - \pi\left[y/6+1\right]^2 dy$$
$$\boxed{V = \pi \int_1^5 (\sqrt{y-5}+1)^2 - (y/6+1)^2 \, dy}$$

$x^2 + 5 = 6x \qquad x = 1, 5 \Rightarrow y = 6, 30$
$x^2 - 6x + 5 = 0$
$(x-5)(x-1) = 0$

or with shells
$$\boxed{V = 2\pi \int_1^5 (x+1)(6x - x^2 - 5)\, dx}$$

6. (8 points) Find the length of the curve $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ for $1 \le x \le 2$.

$$L = \int_a^b \sqrt{1 + (y')^2}\, dx$$
$y' = \frac{1}{2}x^2 - \frac{1}{2}x^{-2}$
$(y')^2 = \frac{1}{4}x^4 - \frac{1}{2} + \frac{1}{4}x^{-4}$
$(y')^2 + 1 = \frac{1}{4}x^4 + \frac{1}{2} + \frac{1}{4}x^{-4}$
$= \left(\frac{1}{2}x^2 + \frac{1}{2}x^{-2}\right)^2$

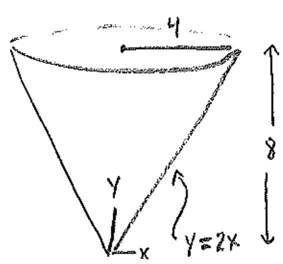
$$L = \int_1^2 \sqrt{\left(\tfrac{1}{2}x^2 + \tfrac{1}{2}x^{-2}\right)^2}\, dx$$
$$= \int_1^2 \tfrac{1}{2}x^2 + \tfrac{1}{2}x^{-2}\, dx$$
because quantity under root root is (+)
$$= \tfrac{1}{6}x^3 - \tfrac{1}{2}x^{-1}\Big|_1^2$$
$$= \frac{8}{6} - \frac{1}{4} - \left(\frac{1}{6} - \frac{1}{2}\right)$$
$$= \frac{7}{6} + \frac{1}{4}$$
$$= \boxed{\frac{17}{12}}$$

7. (8 points) Let $f(x) = \sqrt{3x - x^2}$ on the interval $[0, 3]$. Find the area of the surface generated is when f is revolved about the x-axis.

$$SA = \int_a^b 2\pi r\, ds = \int_a^b 2\pi r \sqrt{1 + (y')^2}\, dx$$
$y' = \frac{1}{2}\,\frac{3-2x}{\sqrt{3x-x^2}}$
$(y')^2 = \frac{1}{4}\,\frac{(3-2x)^2}{3x-x^2}$

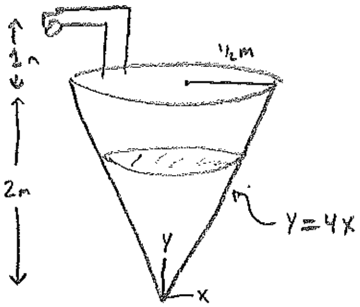
$$SA = \int_0^3 2\pi\sqrt{3x-x^2}\sqrt{1 + \frac{1}{4}\frac{(3-2x)^2}{3x-x^2}}\, dx$$
$$= \int_0^3 2\pi \sqrt{\frac{4(3x-x^2) + (3-2x)^2}{4}}\, dx$$
$$= \int_0^3 2\pi \sqrt{9/4}\, dx$$
$$= \int_0^3 3\pi\, dx$$
$$= \boxed{9\pi}$$

8. (8 points) Find the surface area of a cone (excluding the base) with radius 4 and height 8 using integration and a surface area integral.



$$SA = \int_0^4 2\pi r\, ds$$
$$= \int_0^4 2\pi r \sqrt{1 + (y')^2}\, dx$$
$$= \int_0^4 2\pi x \sqrt{1 + (2)^2}\, dx$$
$$= \int_0^4 2\pi\sqrt{5}\, x\, dx$$
$$= \pi\sqrt{5}\, x^2\Big|_0^4$$
$$= \boxed{16\pi\sqrt{5}}$$

9. (6 points) An inverted conical tank (point down) is 2m hight and has a base radius of $\frac{1}{2}$m. If the tank is full of a liquid with density, ρ, how much work is required to pump the water to a level 1m above the top of the tank. Your answer can be in terms of ρ and g. Set up the integral, but do not evaluate.



$$W = \int_a^b \rho g A D\,dy$$

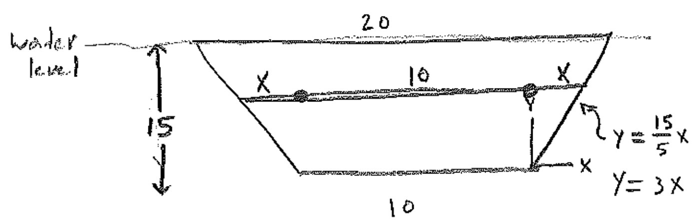
$$= \int_0^2 \rho g A D\,dy$$

$$= \int_0^2 \rho g \pi r^2 D\,dy$$

$$= \int_0^2 \rho g \pi x^2 D\,dy$$

$$= \boxed{\int_0^2 \rho g \pi \left(\frac{y}{4}\right)^2 (3-y)\,dy}$$

10. (6 points) The face of a dam is shaped like a trapezoid. The length of the base is 10m, the length of the top is 20m with a height of 15m. If the dam is full of water, find the total force on the face of the dam. Set up the integral, but do not evaluate.



$$F = \int_a^b \rho g\, l\, D\,dy$$

$$= \int_0^{15} \rho g (10+2x)(15-y)\,dy$$

$$= \boxed{\int_0^{15} \rho g \left(10+\tfrac{2}{3}y\right)(15-y)\,dy}$$

11. (8 points) Evaluate $\displaystyle\int_1^4 \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx$.

$u = \sqrt{x}$

$du = \frac{1}{2}\frac{1}{\sqrt{x}}\,dx$

$$20\int_{x=1}^{x=4} 10^u\,du = 20\cdot 10^u \ln 10\Big|_{x=1}^{x=4} = 20\ln(10)\,10^{\sqrt{x}}\Big|_1^4 = 2000\ln(10) - 200\ln(10)$$

$$= \boxed{1800\ln(10)}$$

12. (8 points) Evaluate $\displaystyle\int_0^{\ln 2} \tanh x\,dx$.

$$= \int_0^{\ln 2} \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$$

$u = e^x + e^{-x}$

$du = e^x - e^{-x}\,dx$

$$= \int_{x=0}^{x=\ln 2} \frac{1}{u}\,du = \ln(e^x + e^{-x})\Big|_0^{\ln 2} = \ln(2+\tfrac{1}{2}) - \ln 2$$

$$= \ln\left(\tfrac{5}{2}\right) - \ln 2$$

$$= \boxed{\ln\left(\tfrac{5}{4}\right)}$$

13. (8 points) Find $H'(x)$ where $H(x) = (x+1)^{2x}$.

$$y = (x+1)^{2x}$$

$$\ln y = \ln\left[(x+1)^{2x}\right]$$

$$\ln y = 2x\ln(x+1)$$

$$\frac{1}{y}y' = 2\ln(x+1) + \frac{2x}{x+1}$$

$$y' = y\left[2\ln(x+1) + \frac{2x}{x+1}\right]$$

$$y' = \boxed{(x+1)^{2x}\left[2\ln(x+1) + \frac{2x}{x+1}\right] = H'}$$

Question	Points	Score
1	8	
2	8	
3	12	
4	6	
5	6	
6	8	
7	8	
8	8	
9	6	
10	6	
11	8	
12	8	
13	8	
Total:	100	