Exam 1 **Name: Solution**

Math 132-02 **Calculus II** **September 14, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 65 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.

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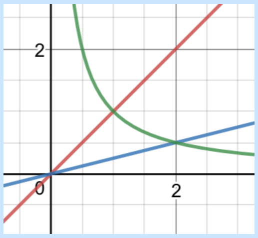
1. (8 points) To be completed once exams are graded and returned. Please correct any problem with points deducted. All corrections should be completed neatly on a separate sheet of paper. Once you have finished your corrections, take your exam and corrections to the Office of Student Learning (OSL), and a tutor will check your answers and sign below. The checked solutions should be given to your instructor.

   Signature: ______________________________
   Print Name: ______________________________
   Date: ________________

Question	Points	Score
1	8	
2	12	
3	18	
4	8	
5	6	
6	8	
7	8	
8	8	
9	12	
10	12	
Total:	100	

2. (12 points) Find the area of the region bounded by $y = \frac{1}{x}$, $y = x$, and $y = \frac{x}{4}$ for $x > 0$.

Solution:

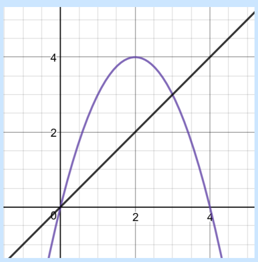


$$\begin{aligned} A &= \int_0^1 \left(x - \frac{x}{4}\right)\,dx + \int_1^2 \left(\frac{1}{x} - \frac{x}{4}\right)\,dx \\ &= \frac{3}{8}x^2\Big|_0^1 + \left(\ln|x| + \frac{x^2}{8}\right)\Big|_1^2 \\ &= \frac{3}{8} + \ln 2 - \frac{3}{8} \\ &= \ln 2 \end{aligned}$$

3. Set up the integral (but do not evaluate) to find the volume of the solid generated when the region R bounded by the parabola $y = 4x - x^2$ and the line $y = x$ is revolved around the

   a. (6 points) y-axis

Solution:



$$\begin{aligned} V &= \int_0^3 2\pi x\left(4x - x^2 - x\right)\,dx \\ &= \int_0^3 2\pi(3x^2 - x^3)\,dx \end{aligned}$$

   b. (6 points) x-axis

Solution:

$$\begin{aligned} V &= \int_0^3 \pi\left(\left(4x - x^2\right)^2 - x^2\right)\,dx \\ &= \int_0^3 \pi\left(16x^2 - 8x^3 + x^4 - x^2\right)\,dx \\ &= \int_0^3 \pi\left(15x^2 - 8x^3 + x^4\right)\,dx \end{aligned}$$

   c. (6 points) line $x = 4$

Solution:

$$V = \int_0^3 2\pi(4-x)\left(4x - x^2 - x\right)\, dx$$
$$= \int_0^3 2\pi(4-x)(3x - x^2)\, dx$$

4. (8 points) Evaluate $\displaystyle\int_1^3 \frac{3^{\ln x}}{x}\, dx$.

Solution:

Let $u = \ln x$, $du = \dfrac{1}{x}\, dx$, so

$$\int_1^3 \frac{3^{\ln x}}{x}\, dx = \int_0^{\ln 3} 3^u\, du = \int_0^{\ln 3} e^{u \ln 3}\, du = \frac{1}{\ln 3} e^{u \ln 3}\Big|_0^{\ln 3} = \frac{1}{\ln 3}\left(e^{\ln 3 \ln 3} - 1\right) = \frac{1}{\ln 3}\left(3^{\ln 3} - 1\right)$$

5. (6 points) Find $\dfrac{d}{dx}\left(1+\dfrac{4}{x}\right)^x$

Solution:

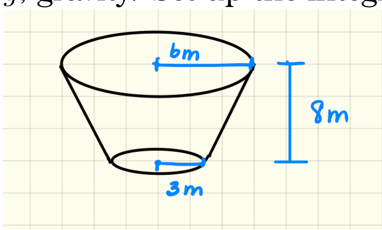
$$\frac{d}{dx}\left(1+\frac{4}{x}\right)^x = \frac{d}{dx}e^{x\ln\left(1+\frac{4}{x}\right)} = e^{x\ln\left(1+\frac{4}{x}\right)}\left(\ln\left(1+\frac{4}{x}\right)+x\cdot\frac{1}{1+\frac{4}{x}}\cdot -\frac{4}{x^2}\right)$$

6. (8 points) Evaluate $\displaystyle\int \frac{\sinh x}{1+\cosh x}\,dx$

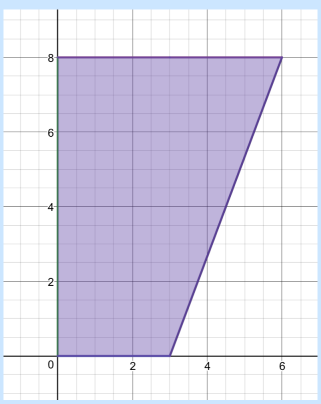
Solution:

Let $u = 1+\cosh x$, $du = \sinh x\,dx$ then $\displaystyle\int \frac{\sinh x}{1+\cosh x}\,dx = \int \frac{1}{u}\,du = \ln|1+\cosh x| + C$

7. (8 points) A tank in the shape shown below has 6 m of water, find the work to empty the tank out of the top. Please leave your answer in terms of ρ, the density of water and g, gravity. Set up the integral, but do not evaluate.



Solution:

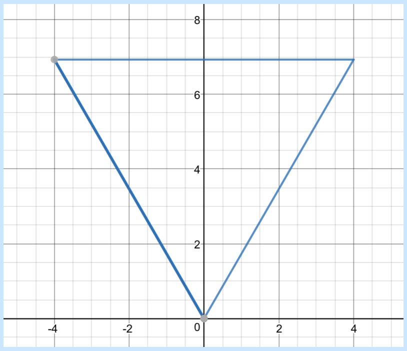


If you rotate the shape shown about the y-axis, you obtain the tank above. The slanted line is $y = \frac{8}{3}(x-3)$ or $x = \frac{3}{8}y+3$ and depth of the water is $0 \le y \le 6$. A slab of water will have the shape of a cylinder, the volume of the cylinder is $V = \pi r^2 dy = \pi\left(\frac{3}{8}y+3\right)^2 dy$ because $r = x$. The depth of the slab $D(y) = 8-y$ so work is

$$W = \int_0^6 \rho g\pi\left(\frac{3}{8}y+3\right)^2(8-y)\,dy.$$

8. (8 points) A trough is filled with a liquid of density 840 $\mathrm{kg/m^3}$. The ends of the trough are equilateral triangles with sides 8 m long and the vertex at the bottom. Find the hydrostatic force on one end of the trough. Set up the integral, but do not evaluate.

Solution:



The slanted side of the triangle is $y = \sqrt{3}x$ or $x = \dfrac{y}{\sqrt{3}}$.
If you create a strip across the shape, the width is $2x = 2\left(\dfrac{y}{\sqrt{3}}\right)$, the depth of the strip is $4\sqrt{3} - y$

$$F = \int_0^{4\sqrt{3}} 2\rho g \left(\frac{y}{\sqrt{3}}\right)(4\sqrt{3} - y)\, dy$$

9. (12 points) The curve $x = \left(1 - y^{2/3}\right)^{3/2}$ for $0 \leq y \leq 1$ is rotated about the x-axis, find the area of the surface generated.

Solution:

Surface area is $S = \int_a^b ds = \int_a^b 2\pi r \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

So $\frac{dx}{dy} = \frac{3}{2}\left(1 - y^{2/3}\right)^{1/2} \cdot \left(-\frac{2}{3}y^{-1/3}\right) = -y^{-1/3}\left(1 - y^{2/3}\right)^{1/2}$ and

$1 + \left(\frac{dx}{dy}\right)^2 = 1 + y^{-2/3}\left(1 - y^{2/3}\right) = 1 + y^{-2/3} - 1 = y^{-2/3}$ then

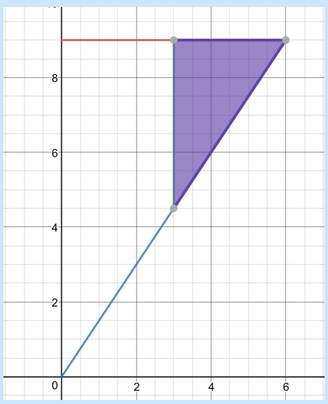
$$\begin{aligned} S &= \int_0^1 2\pi \left(1 - y^{2/3}\right)^{3/2} \sqrt{y^{-2/3}}\, dy \\ &= \int_0^1 2\pi \left(1 - y^{2/3}\right)^{3/2} y^{-1/3}\, dy \\ &= \int_1^0 -\frac{3}{2} \cdot 2\pi (u)^{3/2} du = -3\pi \cdot \frac{2}{5} u^{5/2}\Big|_1^0 = -\frac{6}{5}\pi(0 - 1) = \frac{6}{5}\pi. \end{aligned}$$

where $u = 1 - y^{2/3}$ and $du = -\frac{2}{3}y^{-1/3}\, dy$

10. (12 points) Find the volume of the solid formed when a hole of radius 3 is drilled symmetrically along the axis of a right circular cone of radius 6 and height 9.

Solution:

To find the volume of what remains of the cone after a hole of radius 3 m is cut from the center is like rotating a triangle with sides $x = 3$, $y = 9$, and $y = \frac{3}{2}x$ about the y-axis.



$$\begin{aligned} V &= \int_3^6 2\pi x \left(9 - \frac{3}{2}x\right)\, dx \\ &= 2\pi \int_3^6 \left(9x - \frac{3}{2}x^2\right)\, dx \\ &= 2\pi \left(\frac{9}{2}x^2 - \frac{1}{2}x^3\right)\Big|_3^6 \\ &= 2\pi \left[\frac{9}{2}(36-9) - \frac{1}{2}(6^3 - 27)\right] \\ &= 2\pi \cdot 27 \left[\frac{9}{2} - \frac{1}{2}(8-1)\right] \\ &= 54\pi \end{aligned}$$

OR

$$\begin{aligned} V &= \int_{9/2}^{9} \pi \left[\left(\frac{2}{3}y\right)^2 - 3^2\right]\, dy \\ &= \pi \int_{9/2}^{9} \left(\frac{4}{9}y^2 - 9\right)\, dy \\ &= \pi \left(\frac{4}{27}y^3 - 9y\right)\Big|_{9/2}^{9} \\ ] &= 54\pi \end{aligned}$$