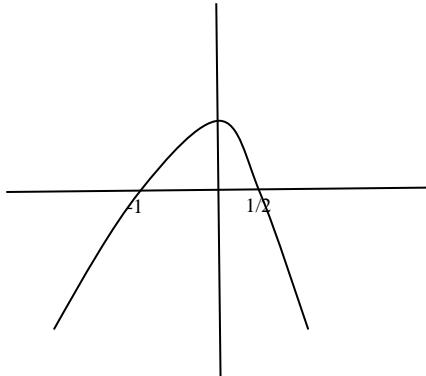


Determine the derivative for each of the following: (Do not simplify!)

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1. $y = \sqrt{3x}$ $\frac{dy}{dx} = \frac{1}{2}(3x)^{-1/2} \cdot 3$
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2. $y = \ln(4x)$ $\frac{dy}{dx} = \frac{1}{4x} \cdot 4$
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3. $y = 3^x$ $\frac{dy}{dx} = 3^x \ln 3$
-
4. $y = \cos(2x)$ $\frac{dy}{dx} = -2\sin(2x)$
-
5. $y = \log_5 6x$ $\frac{dy}{dx} = \frac{1}{6x \ln 5} \cdot 6$
-
6. $y = e^{-x}$ $\frac{dy}{dx} = -e^{-x}$
-
7. $y = \sinh(1-2x)$ $\frac{dy}{dx} = -2\cosh(1-2x)$
-
8. $y = \pi$ $\frac{dy}{dx} = 0$
-
9. $y = \sin x^3$ $\frac{dy}{dx} = 3x^2 \cos(x^3)$
-
10. $y = \tan(2\sqrt{x})$ $\frac{dy}{dx} = \sec^2(2\sqrt{x}) \cdot 2 \cdot \frac{1}{2} x^{-1/2}$
-
11. $y = 1/(2-3x)$ $\frac{dy}{dx} = -(2-3x)^{-2} (-3)$
-
12. $y = \sqrt[3]{5x-1}$ $\frac{dy}{dx} = \frac{1}{3}(5x-1)^{-2/3} \cdot 5$
-
13. $y = \ln x^3$ $\frac{dy}{dx} = \frac{1}{x^3} \cdot 3x^2$
-
14. $y = 2^{x^2}$ $\frac{dy}{dx} = 2^{x^2} \ln 2 \cdot 2x$
-
15. $y = \arctan(6x)$ $\frac{dy}{dx} = \frac{1}{1+(6x)^2} \cdot 6$
-
16. $y = \sin^2 2x$ $\frac{dy}{dx} = 2\sin 2x \cdot \cos 2x \cdot 2$
-
17. $y = \sec(\pi x)$ $\frac{dy}{dx} = \pi \sec(\pi x) \tan(\pi x)$
-
18. $y = \arcsin \sqrt[4]{x}$ $\frac{dy}{dx} = \left(1 - (\sqrt[4]{x})^2\right)^{-1/2} \cdot \frac{1}{4} x^{-3/4}$
-
19. $y = e^{x^{1/2}}$ $\frac{dy}{dx} = e^{x^{1/2}} \cdot \frac{1}{2} x^{-1/2}$
-
20. $y = (7x^2 + 2)^{3/2}$ $\frac{dy}{dx} = \frac{3}{2}(7x^2 + 2)^{1/2} \cdot 14x$
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21. Find $\frac{dy}{dx}$ for $y = e^{-x} \ln(3x)$
- $$\frac{dy}{dx} = \frac{e^{-x}}{x} - e^{-x} \ln(3x)$$
22. Find $\frac{dy}{dx}$ for $y = \frac{\cosh(x^2 + 2)}{x^3 + 4x^2 + 1}$
- $$\frac{dy}{dx} = \frac{2x \sinh(x^2 + 2)(x^3 + 4x^2 + 1) - \cosh(x^2 + 2)(3x^2 + 8x)}{(x^3 + 4x^2 + 1)^2}$$
23. Find $\frac{dy}{dx}$ for $y = \tan^3 \sqrt{x^2 + 2x}$
- $$\frac{dy}{dx} = 3 \tan^2 \sqrt{x^2 + 2x} \cdot \sec^2 \sqrt{x^2 + 2x} \cdot \frac{1}{2} (x^2 + 2x)^{-1/2} (2x + 2)$$
24. Find $\frac{dy}{dx}$ for $y = \frac{e^{-x}}{\tanh x}$
- $$\frac{dy}{dx} = \frac{-e^{-x} \tanh x - e^{-x} \operatorname{sech}^2 x}{\tanh^2 x}$$
25. Find $f'(x)$ for $f(x) = 2^x \tan x + 2x$
- $$f'(x) = e^{x \ln 2} (\ln 2) \tan x + 2^x \sec^2 x + 2$$
26. Find $\frac{dy}{dx}$ for $y = \sqrt{x} \tan^{-1} x$
- $$\frac{dy}{dx} = \frac{\tan^{-1} x}{2\sqrt{x}} + \frac{\sqrt{x}}{1+x^2}$$
27. Find $f'(x)$ for $f(x) = 2x \sinh^2 x$.
- $$f'(x) = 2 \sinh^2 x + 2x(2 \sinh x \cosh x)$$
28. Find the derivative of $f(x) = |2x+1|$
- $$f'(x) = \begin{cases} 2x+1 & \text{if } x \geq -\frac{1}{2} \\ -(2x+1) & \text{if } x < -\frac{1}{2} \end{cases}$$
29. Find $\frac{dy}{dx}$ for $y = (1-x)^x$ for $x < 1$
- $$\frac{dy}{dx} = (1-x)^x \left[\ln(1-x) - \frac{x}{1-x} \right]$$
30. Find $\frac{dy}{dx}$ for $\ln(xy^2) = x^2 + y^2$
- $$\frac{dy}{dx} = \frac{2x - 1/x}{2/y - 2y}$$
31. For $y = f(x)$ shown below, sketch a possible graph of $f'(x)$.



32. Find the derivative of $f(x) = \frac{1}{x+2}$, using the definition of the derivative.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{x+2 - (x+h+2)}{(x+h+2)(x+2)} \right) = \lim_{h \rightarrow 0} \frac{-1}{(x+2)(x+h+2)} = \frac{-1}{(x+2)^2} \end{aligned}$$

33. Find $f''(x)$ where $f(x) = x \sin(6x)$. $f''(x) = 12\cos(6x) - 36x\sin(6x)$

34. At which points on the curve $y = x + 2\sin x$, $0 \leq x \leq 2\pi$ is the tangent line horizontal?

$$y' = 1 + 2\cos x = 0, \text{ when } x = \frac{2\pi}{3} \text{ or } x = \frac{4\pi}{3}$$

35. Find an equation of the tangent line to the curve $\sqrt[3]{x} + \sqrt[3]{y^4} = 2$ at $(1,1)$.

$$\sqrt[3]{x} + \sqrt[3]{y^4} = 2 \Rightarrow \frac{1}{3}x^{-2/3} + \frac{4}{3}y^{1/3}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\frac{1}{3}x^{-2/3}}{\frac{4}{3}y^{1/4}}, \text{ so } m_r = \left. \frac{dy}{dx} \right|_{(1,1)} = -\frac{1}{4} \text{ so the tangent line is}$$

$$y - 1 = -\frac{1}{4}(x - 1)$$

36. Answer the following, using the table below to find:

X	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
-3	0	3	1	0
-2	1	2	2	-3
-1	3	0	-1	-2
0	2	3	-2	3
1	0	-1	-2	-2
2	-2	-2	-1	0
3	-3	0	0	1

a. $f'(3)$ if $f(x) = 5g(x) - 4h(x)$.

$$f'(x) = 5g'(x) - 4h'(x) \text{ so } f'(3) = 5g'(3) - 4h'(3) = -4$$

b. $f'(2)$ if $f(x) = \frac{2g(x)}{h(x)}$.

$$f'(x) = \frac{2g'(x)h(x) - 2g(x)h'(x)}{(h(x))^2} \text{ so } f'(2) = \frac{2g'(2)h(2) - 2g(2)h'(2)}{(h(2))^2} = 1$$

c. $f'(-2)$ if $f(x) = g(h(x))$.

$$f'(x) = g'(h(x))h'(x) \text{ so } f'(-2) = g'(h(-2))h'(-2) = 3$$

d. $f'(-1)$ if $f(x) = \sqrt{g(x)}$.

$$f'(x) = \frac{1}{2} (g(x))^{-1/2} g'(x) \text{ so } f'(-1) = \frac{1}{2} (g(-1))^{-1/2} g'(-1) = -\frac{1}{2\sqrt{3}}$$

e. If $f(x) = h(x^2 g(x))$, find an equation of the tangent line at $x = -1$.

$$y_0 = f(-1) = h((-1)^2 g(-1)) = h(3) = 0$$

$$f'(x) = h'(x^2 g(x))(2xg(x) + x^2 g'(x)) \text{ so}$$

$$m_{\tan} = f'(-1) = h'((-1)^2 g(-1))(2(-1)g(-1) + (-1)^2 g'(-1)) = -7$$

A tangent line is $y - 0 = -7(x + 1)$

37. Find $f(x)$ if $f'(x) \frac{3x^2}{\sqrt{x^3 + 1}}$ where $f(2) = 6$.

$$f(x) = 2(x^3 + 1)^{1/2} + C, \text{ Now if } f(2) = 6, C = 0 \text{ so } f(x) = 2(x^3 + 1)^{1/2}$$