1. Write an equation of the line that passes through $(-3,7)$ and is perpendicular to the line with equation $y-2x=10$

   Solution: The line: $y=2x+10$ has slope $m_1=2$ so the slope of the perpendicular line is $m_2=-\dfrac{1}{2}$ and the equation of the line is $y-7=-\dfrac{1}{2}(x+3)$

2. Simplify each of the following:

   a. $$\frac{\dfrac{1}{2+h}-\dfrac{1}{2}}{h}$$

   Solution $\dfrac{-1}{2(2+h)}$

   b. $$\frac{x^{-2/5}\sqrt{4x}}{\sqrt[3]{x}}$$

   Solution: $2x^{-7/30}$

3. Solve the inequality $\dfrac{x^2-9}{x-1}\le 0$; express the domain in interval notation.

   Solution: $(-\infty,-3]\cup(1,3]$

4. For the function $f(x)=3x-6$, find $\dfrac{f(x+h)-f(x)}{h}$.

   Solution, $\dfrac{3(x+h)-6-(3x-6)}{h}=\dfrac{3h}{h}=3$

5. Determine each of the following (if they exist) : **Solution**

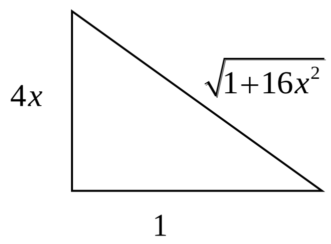
   a. $\sin(0)=0$

   b. $\cos(0)=1$

   c. $\sin^{-1}(4)$ does not exist

   d. $\tan^{-1}(1)=\dfrac{\pi}{4}$

6. Write $\sin\left(\arctan 4x\right)$ as an algebraic function that does not involve a trigonometric function or inverse trigonometric function.



Solution: $\dfrac{4x}{\sqrt{1+16x^2}}$

7. Sketch a graph of each of the following. Label any important points or features.
   a. $\sin x$
   b. $\sqrt{x-3}$ **Check your solution with your graphing calculator.**

8. Give the formal definition of the limit statement $\lim\limits_{x\to c} f(x) = L$. Illustrate the definition with a graph.

   Solution: $\lim\limits_{x\to c} f(x) = L$ means that for all $\varepsilon > 0$, there exists a $\delta > 0$ such that if $x \in (c-\delta, c)\cup(c, c+\delta)$ then $f(x)\in(L-\varepsilon, L+\varepsilon)$. Check your sketch by looking at graphs on page 91.

9. Evaluate each of the following and ***state which limit laws were used***:

a. $$\lim_{x\to 1}\left(\frac{1+3x}{1+4x^2+3x^4}\right)^3 \overset{3,4,6,7}{=} \left(\frac{\lim\limits_{x\to1}1+3\lim\limits_{x\to1}x}{\lim\limits_{x\to1}1+4\left(\lim\limits_{x\to1}x\right)^2+3\left(\lim\limits_{x\to1}x\right)^4}\right)^3 \overset{1,2}{=} \left(\frac{1+3(1)}{1+4(1)^2+3(1)^4}\right)^3 = \frac{1}{8}$$

b. $$\lim_{x\to 2}\frac{x^2+2x-8}{x^4-16} = \lim_{x\to2}\frac{(x+4)(x-2)}{(x^2+4)(x+2)(x-2)} \overset{10}{=} \lim_{x\to2}\frac{x+4}{(x^2+4)(x+2)} \overset{4,5,6}{=} \frac{3}{16}$$

c. $$\lim_{x\to0}\frac{1-\sqrt{1-x^2}}{x} = \lim_{x\to0}\frac{1-\sqrt{1-x^2}}{x}\cdot\frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} = \lim_{x\to0}\frac{1-(1-x^2)}{x\left(1+\sqrt{1-x^2}\right)}$$
$$= \lim_{x\to0}\frac{x^2}{x\left(1+\sqrt{1-x^2}\right)} \overset{10}{=} \lim_{x\to0}\frac{x}{1+\sqrt{1-x^2}} \overset{6,8,4}{=} \frac{0}{1} = 0$$

d. $$\lim_{x\to8^-}\frac{|x-8|}{x-8} = \lim_{x\to8^-}\frac{-(x-8)}{x-8} = \lim_{x\to8^-}(-1) = -1$$

e. $$\lim_{x\to\infty}\frac{\sqrt{x^2-9}}{2x-6} = \lim_{x\to\infty}\frac{|x|\sqrt{1-\frac{9}{x^2}}}{x\left(2-\frac{6}{x}\right)} \overset{4,6,8,10}{=} \frac{\sqrt{\lim\limits_{x\to\infty}1-\lim\limits_{x\to\infty}\frac{9}{x^2}}}{\lim\limits_{x\to\infty}2-\lim\limits_{x\to\infty}\frac{6}{x}} \overset{13}{=} \frac{\sqrt{1}}{2} = \frac{1}{2}$$

f. $\lim\limits_{x\to3} f(x) \Rightarrow \lim\limits_{x\to3^-} 2(x+1) = 8$ and $\lim\limits_{x\to3^-}(x^2-1) = 8$ so $\lim\limits_{x\to3} f(x) = 8$

10. Use the Intermediate Value Theorem to show that $x^4 + x - 3 = 0$ on $(1,2)$.

Solution: $f(x) = x^4 + x - 3$ is continuous on $[1,2]$ because it is a polynomial and $f(1) = -1$ and $f(2) = 15$ so since $f(1) < 0 < f(2)$ then by the IVT there is an $x \in (1,2)$ such that $x^4 + x - 3 = 0$

11. Use the definition of continuity to find constant c to such that $f(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$ is continuous at $x = 4$.

Solution: Definition of Continuity $\lim\limits_{x \to 4^-} f(x) = \lim\limits_{x \to 4^+} f(x) = f(4)$ Now $f(4) = 4c + 20$ and $\lim\limits_{x \to 4^-}\left(x^2 - c^2\right) = 16 - c^2$ and $\lim\limits_{x \to 4^+}(cx + 20) = 4c + 20$ For $f(x)$ to be continuous at $x = 4$,

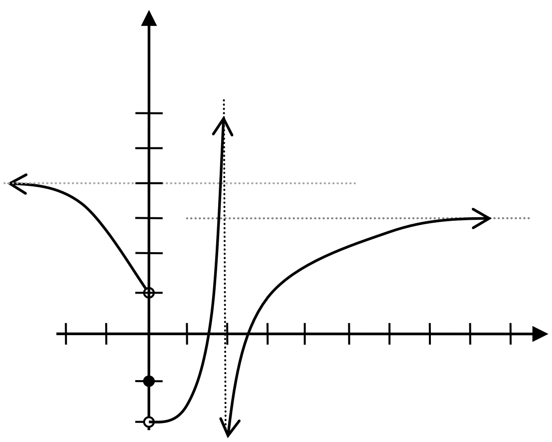
$$16 - c^2 = 4c + 20 \Rightarrow 0 = c^2 + 4c + 4 \Rightarrow 0 = (c+2)^2 \Rightarrow c = -2$$

12. Sketch the graph of an example of a function f that satisfies the following conditions.

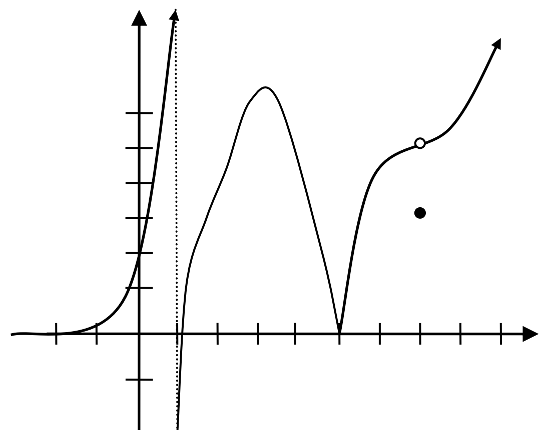
$$\lim_{x \to 0^+} f(x) = -2 \qquad \lim_{x \to 0^-} f(x) = 1 \qquad f(0) = -1$$

$$\lim_{x \to 2^+} f(x) = -\infty \qquad \lim_{x \to 2^-} f(x) = +\infty$$

$$\lim_{x \to \infty} f(x) = 3 \qquad \lim_{x \to -\infty} f(x) = 4$$



13. Using the graph of $f(x)$, answer each of the following:



a. $\lim_{x\to 7} f(x) = 5$

b. $\lim_{x\to 1^-} f(x) = +\infty (DNE)$

c. Where is $f(x)$ not continuous?
Not continuous at $x = 1$ or $x = 7$

d. Where is $f(x)$ not differentiable
**Not on this test!!!

Topics:

- ✓ Review algebra and trig from chapter 1:
    - o Lines
    - o Inverse functions
    - o Trig functions /Inverse Trig functions
    - o Exponential and Logarithmic Functions
- ✓ Limits:
    - o Average rate of change and instantaneous rate of change
    - o Graphical, numerical and algebraic techniques for finding limits; be sure you know how to use limit laws.
    - o Infinite limits; when *y* becomes arbitrarily large in magnitude as *x* approaches *a*.
    - o End behavior; what happens to *y* as *x* becomes arbitrarily large in magnitude.
    - o Continuity
- ✓ Tangent Lines

For additional problems, check out the review problems for Chapters 1 and 2. Note the questions above are simply a sample of questions possible for the exam; it is possible that other types of questions may appear on your exam.