1. Determine the derivative for each of the following:

a. $y=(2x+3)^6$ $\qquad dy/dx = 6(2x+3)^5 \cdot 2$

b. $y=\sec\sqrt{x}$ $\qquad dy/dx = \sec\sqrt{x}\tan\sqrt{x} \cdot \frac{1}{2}x^{-1/2}$

c. $y=\sqrt{\ln x}$ $\qquad dy/dx = \frac{1}{2}[\ln(x)]^{-1/2} \cdot \frac{1}{x}$

d. $y=2^{3x}$ $\qquad dy/dx = 2^{3x} \cdot \ln(2) \cdot 3$

e. $y=e^{x^2-x}$ $\qquad dy/dx = e^{x^2-x} \cdot (2x-1)$

f. $y=\sin(\sin x)$ $\qquad dy/dx = \cos(\sin(x)) \cdot \cos(x)$

g. $y=\cos(x^2-3x+1)$ $\qquad dy/dx = -\sin(x^2-3x+1) \cdot (2x-3)$

h. $y=e^2$ $\qquad dy/dx = 0$

i. $y=\log_2 x^2$ $\quad \frac{\ln x^2}{\ln(2)}$ $\qquad dy/dx = \frac{1}{\ln(2)} \cdot \frac{1}{x^2} \cdot 2x$

j. $y=\left(x+\frac{6}{x}\right)^5$ $\qquad dy/dx = 5(x+6/x)^4 \cdot (1-6x^{-2})$

k. $y=\cosh^4 x$ $\qquad dy/dx = 4(\cosh(x))^3 \cdot \sinh(x)$

l. $y=\sin^{-1}\left(\frac{1}{x}\right)$ $\qquad dy/dx = \frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot (-x^{-2})$

m. $y=\ln(x^2+4)^2$ $\qquad dy/dx = \frac{1}{(x^2+4)^2} \cdot 2(x^2+4) \cdot 2x$

n. $y=\tan\left(\frac{2}{x}\right)$ $\qquad dy/dx = \sec^2(2/x) \cdot -2x^{-2}$

o. $y=\arctan(e^x)$ $\qquad dy/dx = \frac{1}{1+e^{2x}} \cdot e^x$

p. $y=\frac{1}{\sqrt{x^2-4}}$ $\qquad dy/dx = -1/2\,(x^2-4)^{-3/2} \cdot 2x$

q. $y=3^{\cos x}$ $\qquad dy/dx = 3^{\cos(x)} \cdot \ln(3) \cdot -\sin(x)$

r. $y=\sqrt{1+\cos x}$ $\qquad dy/dx = \frac{1}{2}(1+\cos(x))^{-1/2} \cdot -\sin(x)$

s. $y=\tan^{-1}(\tan x)$ $\qquad dy/dx = \frac{1}{1+\tan^2(x)} \cdot \sec^2(x)$

t. $y=\ln(\tan x)$ $\qquad dy/dx = \frac{1}{\tan(x)} \cdot \sec^2(x)$

2. Find dy/dx for $y=x^2\sin(\pi x+1)$.

$y' = 2x\sin(\pi x+1) + x^2\cos(\pi x+1) \cdot \pi$

3. Find an equation for the tangent line at $x=-1$ for the function $f(x)=\frac{2x}{1+x^2}$.

$m(-1) = \frac{2(1+x^2) - 2x(2x)}{(1+x^2)^2} = \frac{4-4}{4} = 0$

$y+1=0 \quad \boxed{y=-1}$

4. Find d^2y/dx^2 for $y=xe^{2x}$.

$y' = e^{2x} + 2x e^{2x}$

$y'' = 2e^{2x} + 2e^{2x} + 4x e^{2x}$

5. Find dy/dx for $y=(x+2)^{1/x}$

$\ln(y) = \frac{1}{x} \ln(x+2)$

$\frac{d}{dx} \ln(y) = \frac{d}{dx} \frac{1}{x} \ln(x+2)$

$\frac{1}{y} \frac{dy}{dx} = \left[-x^{-2}\ln(x+2) + 1/x \cdot \frac{1}{x+2}\right]$

$\frac{dy}{dx} = \left[-x^{-2}\ln(x+2) + \frac{1}{x} \cdot \frac{1}{x+2}\right] \cdot y$

$\frac{dy}{dx} = \left[-x^{-2}\ln(x+2) + \frac{1}{x} \cdot \frac{1}{x+2}\right] \cdot (x+2)^{1/x}$

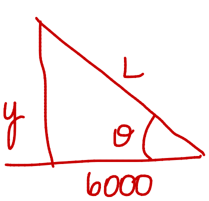
6. Find dy/dx for $xy^2+x^2y-2=0$.

$\frac{d}{dx}\left[xy^2 + x^2y - 2\right] = \frac{d}{dx} 0$

$y^2 + x2y\frac{dy}{dx} + 2xy + x^2\frac{dy}{dx} - 0 = 0$

$\frac{dy}{dx} = \frac{-y^2 - 2xy}{2xy + x^2}$

7. At a distance of 6000 ft from the launch site, a spectator is observing a space shuttle being launched. If the space shuttle lifts off vertically, at what rate is the distance between the spectator and shuttle changing at the instant when the angle of elevation is $\pi/6$ and the shuttle is traveling at 880 ft/sec?



$\tan\theta = y/6000 \qquad y = 6000\tan\theta$

$\cos\theta = \frac{6000}{L} \qquad L = 6000\sec\theta$

$x^2 + y^2 = L^2$

$2x\cancel{\frac{dx}{dt}}^{0} + 2y\frac{dy}{dt} = 2L\frac{dL}{dt}$

$\frac{dL}{dt} = \frac{y}{L}\frac{dy}{dt} = \frac{\tan\theta}{\sec\theta}\frac{dy}{dt} = \sin\theta\left(\frac{dy}{dt}\right) =$

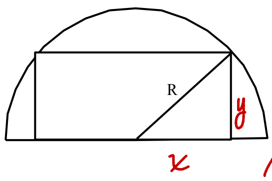
$\frac{dL}{dt} = \sin\pi/6\,(880) = \frac{1}{2}(880) = \underline{440 \text{ ft/s}}$

8. If $y=\frac{1}{x}$, use the definition of derivative to find $\frac{dy}{dx}$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\lim\frac{1}{h}\left[\frac{x-(x+h)}{x(x+h)}\right]$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{-h}{x(x+h)}\right]=\frac{-1}{x^2}$$

9. Find the rectangle of largest area that can be inscribed in a semicircle of radius R, assuming one side of the rectangle lies on the diameter of the semicircle as shown.



$$A=2xy=x\sqrt{R^2-x^2}$$

$$x^2+y^2=R^2$$

$$y=\sqrt{R^2-x^2}$$

$$A'=2\left[\sqrt{R^2-x^2}-\frac{x(-2x)}{2\sqrt{R^2-x^2}}\right]=\frac{R^2-2x^2}{\sqrt{R^2-x^2}}=0\Rightarrow$$

$$\boxed{x=\frac{R}{\sqrt{2}}\quad y=\frac{R}{\sqrt{2}}}$$ ← gives max area

10. If $f(x)=\frac{3-x}{x}$, find $(f\circ f)(x)$. What is the domain of $(f\circ f)(x)$?

$$f(f(x))=\frac{3-\left(\frac{3-x}{x}\right)}{\left(\frac{3-x}{x}\right)}=\frac{4x-3}{x}\cdot\frac{x}{3-x}=\frac{4x-3}{3-x}$$

domain all reals except $x=0$ & $x=3$

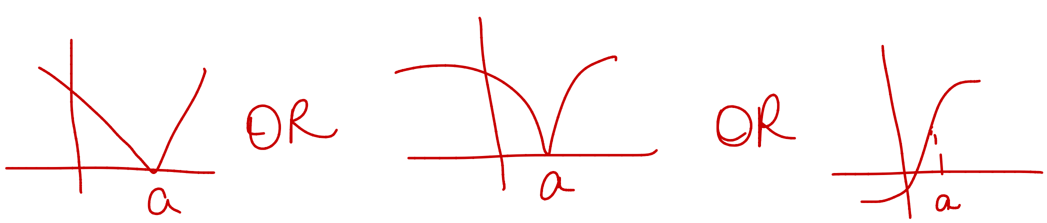
11. Are the lines $2x+y=1$ and $2x-y=1$ perpendicular?

$y=-2x+1$ $\quad$ $y=2x-1$ $\quad$ no b/c $m_1m_2=-4\neq-1$.

12. True or False: If $\lim_{x\to a}f(x)=L$ then $f(a)=L$.

13. If $f(x)$ and $g(x)$ are differentiable, then $\frac{d}{dx}(f(x)g(x))=$ $f'g+g'f$.

14. Sketch a function f where f is continuous at $x=a$ but f is not differentiable at $x=a$.



$h(-3) = f(5) = 3$

15. If $h(x) = f\,[g(x)]$, $g(-3) = 5$, $g'(-3) = 2$, $f(5) = 3$, and $f'(5) = -3$, find an equation of the tangent line to the graph of $h(x)$ at $x = -3$.

$h'(x) = f'(g(x))\,g'(x)$ $\quad h'(-3) = f'(g(-3))\,g'(-3) = f'(5)\,2 = -3\cdot 2 = \underline{\underline{-6}}$

$$\boxed{y - 3 = -6(x+3)}$$

16. Find the maximum and minimum values of $f(x) = Ax + B$ where $A > 0$ and B are constants on $[a, b]$.

no CP.

$f' = A$ so max & min occur at end pts

since $A > 0$ (positive)

min at $x = a$ max at $x = b$

17. Find $\lim_{x \to 0} \dfrac{\sin 2x}{x} =$ _____.

$= 2 \lim_{x \to 0} \dfrac{\sin 2x}{2x} = 2(1) = 2.$ OR use L'Hop

18. Does the function $g(x) = |x - 2|$ satisfy the hypotheses of the Mean Value Theorem on [1,4].

No $g'(x)$ does not exist at $x = 2$.

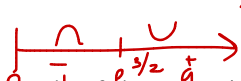
19. By the __Intermediate.__ theorem, if f is continuous on the interval [a, b] and K is between $f(a)$ and $f(b)$ then $K = f(c)$ for some c in (a, b).

20. Let $f(x) = \dfrac{\ln x}{x}$. $x > 0$ Calculate $f''(x)$ and use it to find intervals where the graph of $f(x)$ is concave up and concave down. Find all inflection points.

$f'(x) = \dfrac{\frac{1}{x}\cdot x - \ln x}{x^2} = \dfrac{1 - \ln x}{x^2}$

$f'' = \dfrac{-\frac{1}{x}(x^2) - (1 - \ln x)\,2x}{x^4}$

$= \dfrac{-3 + 2\ln x}{x^3} = 0$ at $x = e^{3/2}$



$\boxed{\begin{array}{l} CD\ (0, e^{3/2}) \\ CU\ (e^{3/2}, \infty) \end{array}}$

21. Find the equation of the tangent line to the graph of the equation $\tan(xy) = y^2$ at $(\pi/4, 1)$.

$\sec^2(xy)\left[1\cdot y + x\dfrac{dy}{dx}\right] = 2y\dfrac{dy}{dx}$

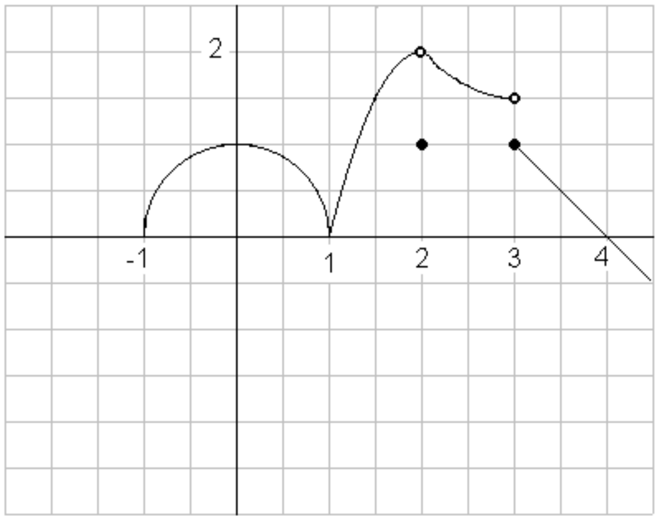
$(x\sec^2 xy - 2y)\dfrac{dy}{dx} = -y\sec^2(xy)$

$\dfrac{dy}{dx} = \dfrac{-y\sec^2(xy)}{x\sec^2 xy - 2y}$

$m = \dfrac{-2}{\frac{\pi}{4}(2) - 2} = \dfrac{-4}{\pi - 4}$

$$\boxed{y - 1 = \frac{-4}{\pi - 4}\left(x - \pi/4\right)}$$

22. Consider the function whose graph is



a. What is the value of $\int_{-1}^{1} f(x)dx$ $= \frac{1}{2}\pi(1)^2 = \pi/2$

b. $f'(x) = 0$ at $x =$ $\underline{0}$.

c. $f''(x) > 0$ for $\underline{2} < x < \underline{3}$

d. $f'(x)$ fails to exist at $x =$ $\underline{1, 2, 3}$.

e. $f(x)$ fails to be continuous at $x =$ $\underline{2, 3}$.

f. $\lim_{x \to x_o} f(x)$ fails to exist at x = $\underline{3}$.

23. Evaluate the following limits

a. $\lim_{x\to 4} \frac{x^3 - 7x^2 + 12x}{4 - x}$ $= \lim_{x\to 4} \frac{x(x-3)\cancel{(x-4)}}{-\cancel{(x-4)}} = \lim_{x\to 4} -x(x-3) = \boxed{-4}$

0/0

b. $\lim_{x\to 0} \frac{e^x - 1}{x^2 + x}$ $\overset{L}{=} \lim_{x\to 0} \frac{e^x}{2x+1} = \frac{1}{1} = 1$

c. $\lim_{x\to 0} \frac{\sin(4x)}{\ln(1+x)}$ $\overset{L}{=} \lim_{x\to 0} \frac{4\cos(4x)}{\frac{1}{1+x}} = 4.$

$\infty \cdot 0$

d. $\lim\limits_{x\to\infty} x\sin\left(\dfrac{\pi}{x}\right) = \lim\limits_{x\to\infty} \dfrac{\sin(\pi/x)}{1/x} \overset{L}{=} \lim\limits_{x\to\infty} \dfrac{\cos\left(\frac{\pi}{x}\right)\left(-\frac{\pi}{x^2}\right)}{-1/x^2} = \lim\limits_{x\to\infty} -\pi\cos\frac{\pi}{x}$ $\boxed{= -\pi}$

$\infty - \infty$

e. $\lim\limits_{x\to 3}\left(\dfrac{1}{x-3} - \dfrac{3}{x^2-3x}\right) = \lim\limits_{x\to 3} \dfrac{\cancel{x-3}}{x(\cancel{x-3})} = \boxed{\dfrac{1}{3}}$

1^∞

f. $\lim\limits_{x\to\pi/2^-} (\sin x)^{\tan x} = \exp\left[\lim\limits_{x\to\pi/2^-} \tan x \ln(\sin x)\right] \overset{L\,0/0}{=} \exp\left[\dfrac{\sin x \ln(\sin x)}{\cos x}\right]$

$= \exp\left[\lim\limits_{x\to\pi/2^-} \dfrac{\cos x \ln(\sin x) + \sin x \cot x}{-\sin x}\right] = e^{0/1} = e^0 = \underline{\underline{1}}$

24. For the graph of $y = \sqrt[3]{x}$, find the inflection point.

$y = x^{1/3}$

$y' = \frac{1}{3}x^{-2/3}$

$y'' = -\frac{2}{9}x^{-5/3}$ $\rightarrow$ $y'' = \dfrac{-2}{9x^{5/3}}$ und at $x=0$

$\cup$ $\cap$

$+$ 0 $-$

$\Rightarrow$ inf pt at $x=0$.

25. Does the graph of $y = \dfrac{2x+\sin x}{x}$ have a vertical asymptote at $x = 0$?

No, $\lim\limits_{x\to 0} \dfrac{2x+\sin x}{x} = \lim\limits_{x\to 0}\left[2 + \dfrac{\sin x}{x}\right] = \underline{\underline{3}} \neq \infty$

26. Verify that the function $f(x) = \sqrt{x}$ satisfies the hypotheses of the Mean Value Theorem on [0, 2].

$f(x) = \sqrt{x}$ is algebraic & cont on $[0,2]$

$f'(x) = \dfrac{1}{2\sqrt{x}}$ is cont on $(0,2) \rightarrow$ so f is diff on $(0,2)$

27. If $f'(x) = \dfrac{1}{2}x^2$ then $f(x) =$ $\underline{\frac{1}{6}x^3 + C}$

28. If $f(x) = -2$ on [-3, 0], then the Riemann Sum for f (x) on the given interval is $-2(0-(-3)) = \underline{\underline{-6}}$.

29. Evaluate the following:

a. $\int\left(x^{2/3}-\frac{2}{x^4}+\frac{1}{x}+\pi^2\right)dx = \frac{3}{5}x^{5/3}-\frac{2x^{-3}}{-3}+\ln|x|+\pi^2 x+C$

b. $\int(\sin 5x+\cos 2x)\,dx = -\frac{1}{5}\cos 5x+\frac{1}{2}\sin 2x+C$

c. $\int \sec(2x)\tan(2x)+\sec^2(5x)\,dx$

$= \frac{1}{2}\sec 2x + \frac{1}{5}\tan 5x + C$

d. $\int\left(e^{-5x}+\frac{1}{\sqrt{1-9x^2}}\right)dx = -\frac{1}{5}e^{-5x}+\frac{1}{3}\arcsin(3x)+C$

e. $\int x^{2/3}(x-1)\,dx = \int\left(x^{5/3}-x^{2/3}\right)dx = \frac{3}{8}x^{8/3}-\frac{3}{5}x^{5/3}+C$

f. $\int\frac{x}{\sqrt{1+x^2}}\,dx$ $\quad u=1+x^2 \quad du=2x\,dx$

$= \frac{1}{2}\int u^{-1/2}\,du = u^{1/2}+C = \sqrt{1+x^2}+C$

g. $\int(\cosh x+\sinh x)\,dx = \sinh x+\cosh x+C$

h. $\int_0^1(3-2x)^5\,dx$ $\quad u=3-2x \quad du=-2dx \quad x=0 \Rightarrow u=3 \quad x=1 \quad u=1$

$-\frac{1}{2}\int_3^1 u^5\,du = -\frac{1}{2}\cdot\frac{u^6}{6}\Big|_3^1 = \frac{-1}{12}\left(1-3^6\right)$

i. $\int_0^1 \frac{5x^2}{2x^3+1}\,dx$ $u=2x^3+1$ $du=6x^2dx$ $x=0$ $u=1$ $x=1$ $u=3$

$= \frac{5}{6}\int_1^3 \frac{1}{u}\,du = \frac{5}{6}\ln|u|\Big|_1^3 = \frac{5}{6}\ln 3$

j. $\int_1^8 \frac{e^{\sqrt[3]{x}}}{x^{2/3}}\,dx$ $u=x^{1/3}$ $du=\frac{1}{3x^{2/3}}dx$

$3\int_1^2 e^u\,du = 3e^u\Big|_1^2 = 3(e^2-e)$

k. $\int_1^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2}\,dx$ $u=\sqrt{x}+1$ $du=\frac{1}{2\sqrt{x}}dx$

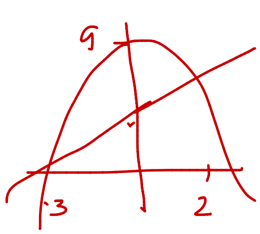
$= 2\int_2^3 u^{-2}\,du = 2\frac{u^{-1}}{-1}\Big|_2^3 = -2\left[\frac{1}{3}-\frac{1}{2}\right]$

30. Evaluate $\int_0^{\frac{3\pi}{4}} |\cos(2x)|\,dx$. $\cos 2x = 0$ at $x=\pi/4, 3\pi/4$

$\int_0^{3\pi/4} |\cos 2x|\,dx = \int_0^{\pi/4}\cos 2x\,dx - \int_{\pi/4}^{3\pi/4}\cos 2x\,dx$

$= \frac{1}{2}\sin 2x\Big|_0^{\pi/4} - \left[\frac{1}{2}\sin 2x\right]_{\pi/4}^{3\pi/4} = \frac{1}{2}(1-0) - \frac{1}{2}(-1-1) = 3/2$

31. Find the area bounded between $y=9-x^2$ and $y=x+3$.



$9-x^2=x+3$

$0=x^2+x-6=(x+3)(x-2)$

$\int_{-3}^{2}\left[(9-x^2)-(x+3)\right]dx = \int_{-3}^{2}(6-x^2-x)\,dx$

$= \left[6x - \frac{1}{3}x^3 - \frac{1}{2}x^2\right]_{-3}^{2} = 12 - \frac{8}{3} - 2 - \left[-18+9-\frac{9}{2}\right] = 10 - \frac{8}{3} + 9 + \frac{9}{2}$

$= \frac{114}{6} - \frac{16}{6} + \frac{27}{6} = \frac{125}{6}.$

32. A tree has been transplanted and after t years of growing at a rate of $\frac{dh}{dt} = 1 + \frac{1}{(t+1)^2}$ ft/yr. At two years it has reached a height of 5 ft. How tall was the tree when it was planted?

$h = \int \frac{dh}{dt}\, dt = t - \frac{1}{t+1} + C \qquad h(2) = 2 - \frac{1}{3} + C = 5 \qquad C = 5 - \frac{5}{3} = \frac{10}{3}$

$h(t) = t - \frac{1}{t+1} + \frac{10}{3} \qquad$ So $h(0) = \underline{7/3}$

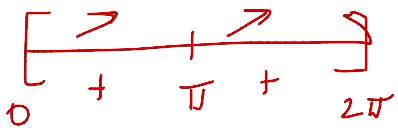
33. For $f(x) = x + \sin x$ for $[0, 2\pi]$

a. Find $f'(x)$. Use it to find critical values of $f(x)$ and intervals where $f(x)$ is increasing and decreasing.

$f' = 1 + \cos x = 0$

$\cos x = -1$

$x = \pi$

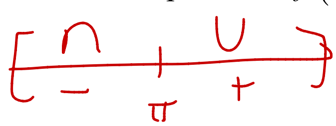


inc on $[0, \pi) \cup (\pi, 2\pi]$

b. Find $f''(x)$. Use it to find inflection points of $f(x)$ and intervals where $f(x)$ is concave up and concave down.

$f''(x) = -\sin x = 0$

$x = \pi$

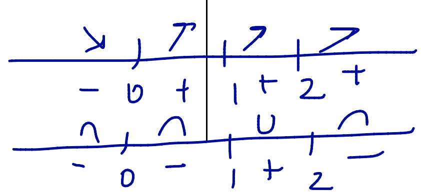


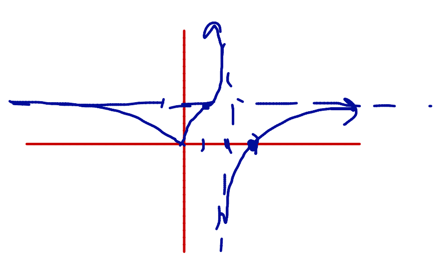
CD $[0, \pi)$

CU $[\pi, 2\pi]$

34. Sketch a possible graph of a function with the following properties.

Domain and Range	First Derivative	Second Derivative
$f(0) = 0$	$f'(x) > 0 \;\; 0 < x < 1,\; 1 < x < 2,\; x > 2$	$f''(x) > 0 \;\; 1 < x < 2$
$f(1) = 1$	$f'(x) < 0 \;\; x < 0$	$f''(x) < 0 \;\; x < 0,\; 0 < x < 1,\; x > 2$
$f(3) = 0$	$\lim_{x \to 0^+} f'(x) = +\infty$	
$\lim_{x \to \pm\infty} f(x) = 1$ HA $y = 1$	$\lim_{x \to 0^-} f'(x) = -\infty$	
$\lim_{x \to 2^-} f(x) = +\infty$		
$\lim_{x \to 2^+} f(x) = -\infty$ VA $x = 2$		





35. Sketch a possible graph of a continuous function $y = f(x)$ using the graph of $f'(x)$ shown below, if $f(0) = f(3) = 0$.