

$$\frac{d}{dx}(x^n) = nx^{n-1} \quad \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Evening Final Exam Review

Math 131

Determine the derivative for each of the following:

1. $y = 5x^4 + \sqrt[3]{x} - \frac{1}{2x^3}$ $\frac{dy}{dx} = 20x^3 + \frac{1}{3}x^{-\frac{2}{3}} - \frac{3}{2}x^{-4}$

2. $y = \ln 6$ $\frac{dy}{dx} = 0$

3. $y = e^{x^3+2x}$ $\frac{dy}{dx} = e^{x^2+2x} \cdot (2x+2)$

4. $y = \tan(x + \sqrt{x})$ $\frac{dy}{dx} = \sec^2(x + \sqrt{x}) \left(1 + \frac{1}{2}x^{-\frac{1}{2}}\right)$

5. $y = \sqrt[4]{7x^4 + 8x^2 + 3} = u^{\frac{1}{4}}$ $\frac{dy}{dx} = \frac{1}{4}(7x^4 + 8x^2 + 3)^{-\frac{3}{4}} \cdot 28x^3 + 16x$

6. $y = 4^x + \log_5 x = \frac{4^x}{\ln 5}$ $\frac{dy}{dx} = 4^x \ln 4 + \frac{1}{\ln 5} \cdot \frac{1}{x}$

7. $y = \cos^6 x = u^6$ $\frac{dy}{dx} = 6(\cos x)^5 (-\sin x)$

8. $y = \ln(4\sqrt[3]{x^2}) = \ln(4x^{\frac{2}{3}})$ $\frac{dy}{dx} = \frac{1}{4x^{\frac{1}{3}}} \cdot 4 \cdot \frac{2}{3}x^{-\frac{1}{3}}$

9. $y = \arctan(x^2)$ $\frac{dy}{dx} = \frac{1}{1+(x^2)^2} \cdot 2x$

10. $y = \sec(5x+1)$ $\frac{dy}{dx} = \sec(5x+1)\tan(5x+1) \cdot 5$

11. $y = \arcsin(\sqrt{x})$ $\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \Rightarrow \frac{1}{2}x^{-\frac{1}{2}}$

12. Find $\frac{dy}{dx}$ for $y = 2x\sqrt{3x-1}(\sin x)$.

13. Find $\frac{dy}{dx}$ for $y = \frac{x}{\cos(e^{10x})}$.

14. Find $\frac{dy}{dx}$ for $y = x^{2x+1}$

15. Find an equation of the line tangent to the curve $x^4 - x^2y + y^4 = 1$ at the point $(-1, 1)$.

16. $\int \left(x^{\frac{5}{3}} - 2x^{\frac{2}{5}} + \frac{2}{x^4} + \pi^3 \right) dx$

17. $\int x^{\frac{1}{3}}(2x^2 - 3x + 1) dx$

18. $\int \sec^2(2x) dx$

19. $\int \sin(\pi x) dx$

$$(12) \quad y = 2x(\sqrt{3x-1} \sin x)$$

$$\frac{dy}{dx} = 2\sqrt{3x-1} \sin x + 2x \left[\frac{1}{2}(3x-1)^{-\frac{1}{2}} \cdot 3 \sin x + \sqrt{3x-1} \cos x \right]$$

$$(13) \quad y = \frac{x}{\cos(e^{10x})}$$

$$\frac{dy}{dx} = \frac{1(\cos(e^{10x}) - x[-\sin(e^{10x}) \cdot e^{10x} \cdot 10])}{[\cos(e^{10x})]^2}$$

$$(14) \quad y = x^{2x+1}$$

$$\ln y = \ln x^{2x+1}$$

$$\ln y = (2x+1) \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = 2 \ln x + (2x+1) \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y \left[2 \ln x + \frac{2x+1}{x} \right]$$

(15)

$$x^4 - x^2y + y^2 = 1 \quad (-1, 1)$$

$$y - y_0 = m(x - x_0)$$

Implicit diff $\rightarrow \frac{dy}{dx} \rightarrow m$ $y(x) \rightarrow$
 add on $\frac{dy}{dx}$

$$4x^3 - \left[2xy + x^2 \frac{dy}{dx} \right] + 2y \frac{dy}{dx} = 0$$

$$4x^3 - 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$(2y - x^2) \frac{dy}{dx} = 2xy - 4x^3$$

$$\frac{dy}{dx} = \frac{2xy - 4x^3}{2y - x^2}$$

$$m = \frac{dy}{dx} \Big|_{(-1,1)} = \frac{-2+4}{4-1} = \boxed{\frac{2}{3}}$$

$$\boxed{y - 1 = \frac{2}{3}(x+1)}$$

$$\textcircled{16} \int \left(x^{5/3} - 2x^{2/5} + \frac{2}{x^4} + \pi^3 \right) dx$$

$$= \int \left(x^{5/3} - 2x^{2/5} + 2x^{-4} + \pi^3 \right) dx$$

$$= \frac{3}{8} x^{8/3} - 2 \cdot \frac{5}{7} x^{7/5} + 2 \cdot \frac{x^{-3}}{-3} + \pi^3 x + C$$

$$\textcircled{17} \int x^{1/3} (2x^2 - 3x + 1) dx$$

$$= \int [2x^{7/3} - 3x^{4/3} + x^{1/3}] dx$$

$$= 2 \cdot \frac{3}{10} x^{10/3} - 3 \cdot \frac{3}{7} x^{7/3} + \frac{3}{4} x^{4/3} + C$$

$$\textcircled{18} \int \sec^2(2x) dx = \frac{1}{2} \tan 2x + C$$

$$\textcircled{19} \int \sin \pi x dx = -\frac{1}{\pi} \cos \pi x + C$$

$$\textcircled{20} \int (3x-8)^8 dx = \frac{1}{3} \cdot \frac{1}{9} (3x-8)^9 + C$$

$$\textcircled{21} \int \frac{1}{4x-1} dx = \frac{1}{4} \ln |4x-1| + C$$

$$u = 4x-1 \quad du = 4dx \rightarrow \frac{1}{4} \int \frac{1}{u} du = \frac{1}{4} \ln |u| + C$$

$$\int f'(g(x))g'(x)dx \quad u=g(x)$$

$$(22) \int e^{\tan \theta} \sec^2 \theta d\theta = e^{\tan \theta} + C$$

$u = \tan \theta \quad \int e^u du = e^u + C$
 $du = \sec^2 \theta d\theta$

$$(23) \int \frac{e^x}{e^x - 1} dx = \ln|e^x - 1| + C$$

$u = e^x - 1 \quad \int \frac{1}{u} du = \ln|u| + C$
 $du = e^x dx$

$$(24) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{7 \sin x}{2+7 \cos x} dx$$

$$u = 2 + 7 \cos x \quad = -\frac{1}{7} \int_9^2 \frac{1}{u} du = -\frac{1}{7} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_9^2$$

$du = -7 \sin x dx$
 $x=0 \quad u = 2 + 7 = 9$
 $x=\frac{\pi}{2} \quad u = 2 + 7(0) = 2$

$$= \boxed{-\frac{2}{21} \left[2^{\frac{3}{2}} - 9^{\frac{3}{2}} \right]}$$

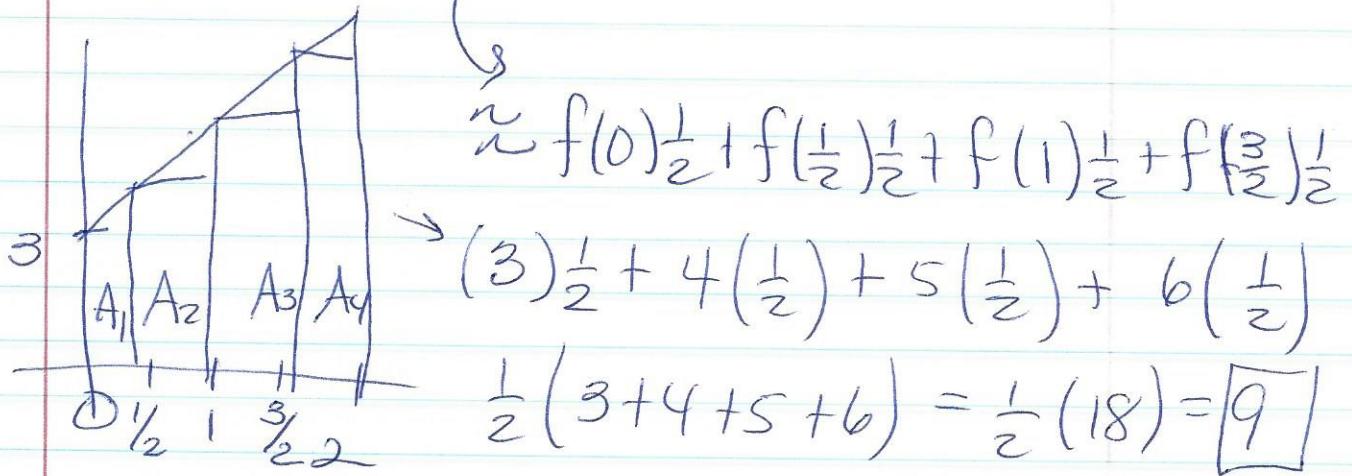
$$(25) \int_0^1 \frac{4x}{2x^2 + 1} dx = \frac{1}{4} \int_1^3 \frac{1}{u} du = \frac{1}{4} \ln|u| \Big|_1^3$$

$$u = 2x^2 + 1 \quad = \frac{1}{4} (\ln 3 - \ln 1)$$

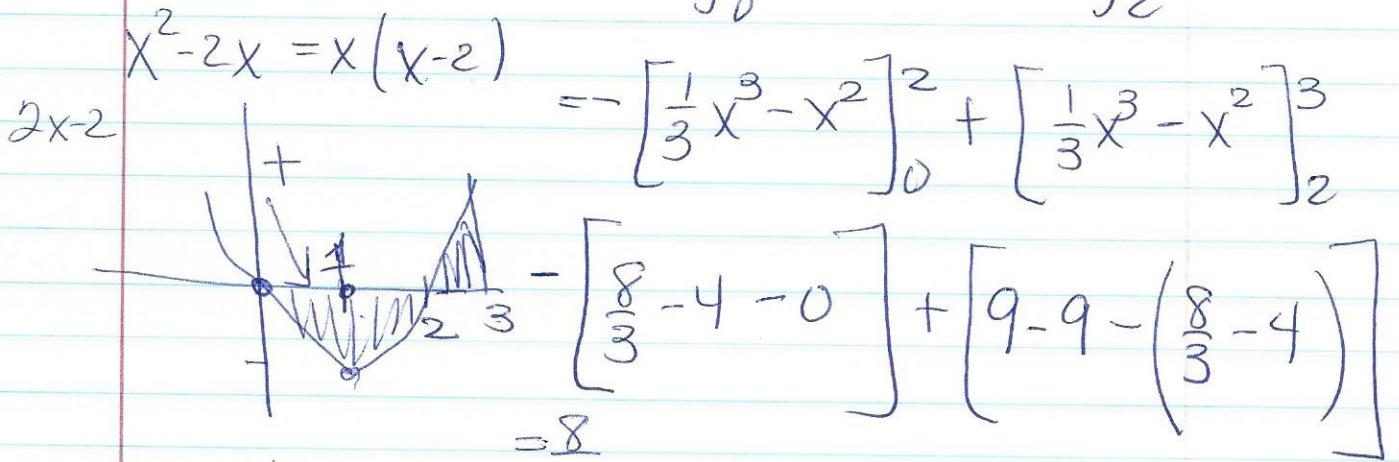
$du = 4x dx$
 $x=0 \quad u=1$
 $x=1 \quad u=3$

$$= \boxed{\frac{1}{4} \ln 3}$$

$$\textcircled{26} \int_0^2 (2x+3) dx \quad \Delta x = \frac{2-0}{4} = \frac{1}{2}$$



$$\textcircled{27} \int_0^3 |x^2 - 2x| dx = \int_0^2 (x^2 - 2x) dx + \int_2^3 (x^2 - 2x) dx$$



$$\textcircled{28} F(x) = \int_0^{x^2} \frac{e^t}{t^2 + 1} dt$$

$$F'(k) = \frac{e^{x^2}}{(x^2)^2 + 1} \cdot 2x$$

(29)

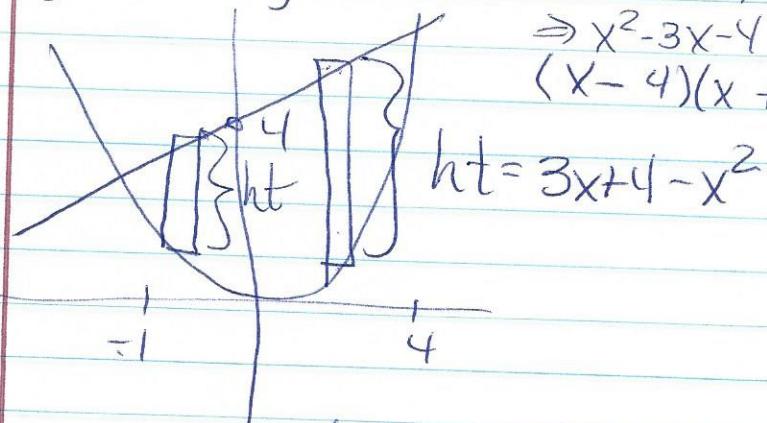
$$y = x^2$$

$$y = 3x + 4$$

$$x^2 = 3x + 4$$

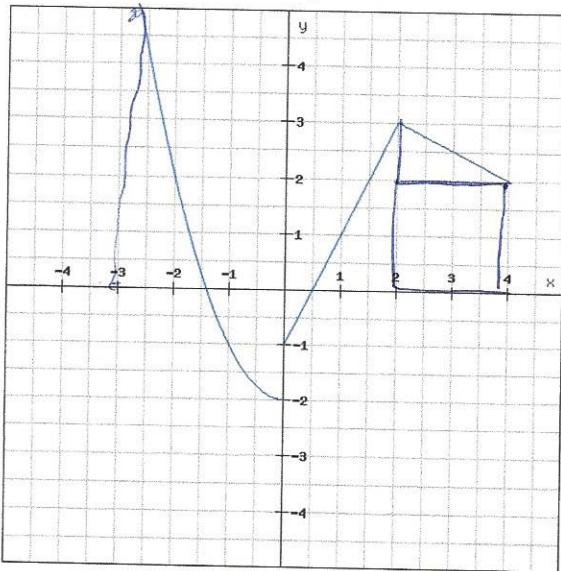
$$\Rightarrow x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$



$$ht = 3x + 4 - x^2$$

$$\begin{aligned}
 A &= \int dt = \int_{-1}^4 (3x + 4 - x^2) dx = \left[\frac{3}{2}x^2 + 4x - \frac{1}{3}x^3 \right]_{-1}^4 \\
 &= \frac{3}{2} [16 - 1] + 4 [4 + 1] - \frac{1}{3} (64 + 1) \\
 &= \boxed{\frac{45}{2} + 20 - \frac{65}{3}}
 \end{aligned}$$



$$y = mx + b$$

$$y' = m$$

30. Use the graph above to answer the following: $\boxed{[-4, 4]}$
- For what values of c does $\lim_{x \rightarrow c} f(x)$ not exist? $c = 0$
 - What is the $\lim_{x \rightarrow 0^-} f(x)$? $\lim_{x \rightarrow 0^+} f(x)$? $\lim_{x \rightarrow 0^-} f(x) = -2$ $\lim_{x \rightarrow 0^+} f(x) = -1$
 - When is $f''(x) > 0$? $(-\infty, 0)$
 - For what values of x is $f'(x) > 0$? $(0, 2)$
 - Evaluate $\int_0^4 f(x) dx = \frac{1}{2}(\frac{1}{2})(-1) + \frac{1}{2}(\frac{3}{2})(3) + 2(2) + \frac{1}{2}(2)(1) = -\frac{1}{4} + \frac{9}{4} + 2 + 1 = \boxed{5}$
 - Compute $f'(1) = m = 2$
 - For what values of x does $f'(x)$ not exist? $x = 0, 2, -3, 4$
31. Verify that $f(x) = x^3 - 3x^2$ satisfies the hypotheses of Rolle's Theorem on $[0, 3]$.
32. Is $f(x) = |x - 1|$ differentiable for all x in $[-5, 5]$?
33. If $y = \pi^5$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.
34. Find $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$
35. Find $\lim_{x \rightarrow 0} (1-2x)^{1/x}$
36. For $f(x) = x + \sin x$ for $[0, 2\pi]$
 - Find $f'(x)$. Use it to find critical values of $f(x)$ and intervals where $f(x)$ is increasing and decreasing.

③ Rolle's Thm $f(x) = x^3 - 3x^2$ $[0, 3]$

1) $f(x)$ cont on $[0, 3]$

Yes b/c f is a poly & they cont everywhere.

2) f is diff on $(0, 3)$

$$f'(x) = 3x^2 - 6x$$

is a poly \rightarrow cont every \rightarrow diff on every where

Yes diff

3) $f(a) = f(b) = 0$

$$f(0) = 0$$

$$f(3) = 27 - 3(9) = 0$$

Yes $f(0) = f(3) = 0$

$\Rightarrow c \in (0, 3)$ such that

$$f'(c) = 0$$

$$3x^2 - 6x = 0 \text{ for some } c$$

$$3x(x-2) = 0$$

$$x=0 \text{ or } (x=2)$$

(32)

$f(x) = |x-1|$ is diff on $[-5, \cancel{1}]$

$$f(x) = \begin{cases} x-1 & x-1 \geq 0 \\ -(x-1) & x-1 < 0 \end{cases}$$



$$f'(x) = \begin{cases} 1 & x > 1 \\ \text{DNE} & x = 1 \\ -1 & x < 1 \end{cases}$$

(33) $y = \pi^5$

$$\frac{dy}{dx} = 0$$

(34)

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} \stackrel{0/0}{=} \lim_{x \rightarrow 0} \frac{\sec^2 x}{1} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x - x \cos x}{x - \sin x} \stackrel{0/0}{=} \lim_{x \rightarrow 0} \frac{\cos x - \cos x + x \sin x}{1 - \cos x} = \frac{0}{0} + 0 = 0$$

$$\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x} \stackrel{0/0}{=} \lim_{x \rightarrow 0} \frac{\sin x + x \cos x}{\sin x} = \frac{0}{0} + 1 = 1$$

$$\lim_{x \rightarrow 0} \frac{\cos x + \cos x - x \sin x}{\cos x} = \frac{2}{1} = \boxed{2}$$

$$\textcircled{35} \lim_{x \rightarrow 0^+} (1-2x)^{1/x} = \boxed{e^{-2}}$$

$$y = (1-2x)^{1/x}$$

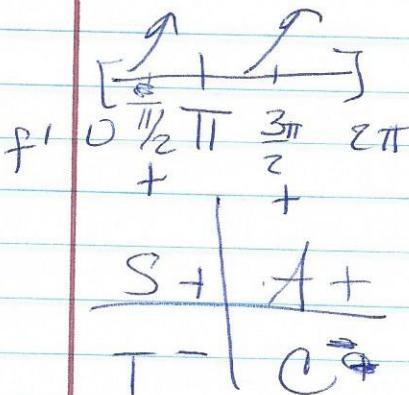
$$\ln y = \frac{1}{x} \ln(1-2x)$$

$$\frac{\partial}{\partial x} \lim_{x \rightarrow 0^+} \frac{\ln(1-2x)}{x} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{1-2x}(-2)}{1} = -2$$

$$\textcircled{36} f(x) = x + \sin x \quad [0, 2\pi]$$

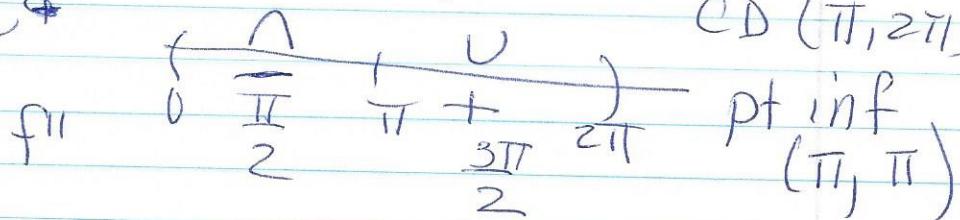
a) $f'(x) = 1 + \cos x = 0$
 $\cos x = -1$

$x = \pi$ f is always inc
 $[0, \pi] \cup (\pi, 2\pi]$



b) $f''(x) = -\sin x = 0$
 $x = 0, \pi, 2\pi$

CU $(0, \pi)$
CD $(\pi, 2\pi)$



pt inf (π, π)

37



$$\frac{dh}{dt} = 0.1 \text{ cm/h} \quad \frac{1 \text{ m}}{100 \text{ cm}} = 0.001 \text{ m/h}$$

$$V = 1 \text{ m}^3 \quad \frac{dV}{dt} = 0$$

Find $\frac{dr}{dt}$ when $r = 8 \text{ m}$.

$$l = \pi(8)^2 h$$

$$h = \frac{1}{\pi 64}$$

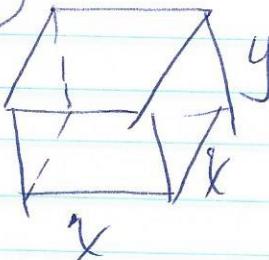
$$V = \pi r^2 h$$

$$0 = 2\pi r \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

$$0 = 2\pi(8) \frac{dr}{dt} \left(\frac{1}{64\pi}\right) + 64\pi(0.001)$$

$$+ 0.064\pi = \frac{1}{4} \frac{dr}{dt} \Rightarrow \boxed{\frac{dr}{dt} = 4(0.064)\pi}$$

38



$$V = 32,000 \text{ cm}^3 = x^2 y$$

$$y = \frac{32,000}{x^2}$$

$$SA = x^2 + 2xy + 2xy$$

$$= x^2 + 4xy$$

$$SA(x) = x^2 + 4x \frac{32,000}{x^2} = x^2 + \frac{128,000}{x}$$

$$SA' = 2x - \frac{128,000}{x^2} = 0 = \frac{2x^3 - 128,000}{x^2} = 0$$

$$2x^3 = 128000$$

$$x^3 = 64,000$$

$$x = \cancel{80}40$$

$$y = \frac{32,000}{1600} = 20$$

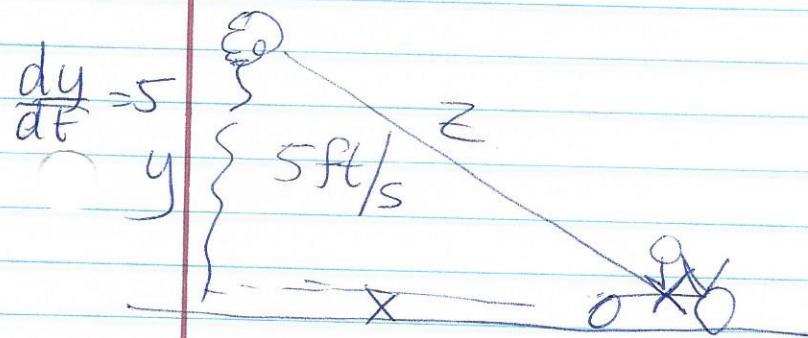
Verify

$$SA'' = 2 + \frac{256000}{x^3}$$

$$SA''(40) > 0 \Rightarrow$$

local min

(39)



$$\frac{dy}{dt} = 5$$

$$\frac{dx}{dt} = 15 \text{ ft/s}$$

$$z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$t=0 \quad y = 45 \text{ ft}$$

$$x = 0$$

$$t=3 \quad y = 45 + 3(5) \\ = 60 \text{ ft}$$

$$x = 0 + 15(3) = 45$$

$$\frac{dz}{dt} = \sqrt{60^2 + 45^2}$$

$$15\sqrt{9+4} = \sqrt{13}$$

$$\frac{dz}{dt} = \frac{1}{z} \left[x \frac{dx}{dt} + y \frac{dy}{dt} \right] = \frac{1}{75} \left[45(15) + 60(5) \right]$$

$$\frac{1}{75} \left[3(15^2) + 4(5^2) \right] \\ 9+4 = \boxed{\sqrt{13}}$$

- b. Find $f''(x)$. Use it to find inflection points of $f(x)$ and intervals where $f(x)$ is concave up and concave down.
37. A circular oil slick of uniform thickness is caused by a spill of 1 m^3 oil. The thickness of the oil is decreasing at a rate of 0.1 cm/h . At what rate is the radius of the slick increasing when the radius is 8 m ?
38. A box with a square base and no top must have a volume of $32,000 \text{ cm}^3$. Find the dimensions of the box that minimize the amount of material used. Verify with the second derivative test.
39. A balloon is rising at a constant speed of 5 ft/s . A boy is cycling along a straight road at a speed of 15 ft/s . When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3s later?

40. Sketch a possible graph of a function with the following properties.

Domain and Range

$$f(0) = 0$$

$$f(1) = 1$$

$$f(3) = 0$$

$$\lim_{x \rightarrow \pm\infty} f(x) = 1$$

$$\lim_{x \rightarrow 2^-} f(x) = +\infty$$

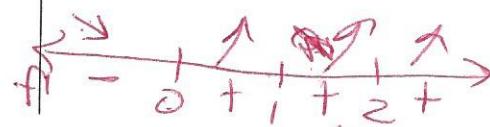
$$\lim_{x \rightarrow 2^+} f(x) = -\infty$$

First Derivative

$$f'(x) > 0 \quad 0 < x < 1, 1 < x < 2, x > 2$$

$$f'(x) < 0 \quad x < 0$$

$$\begin{cases} \lim_{x \rightarrow 0^+} f'(x) = +\infty \\ \lim_{x \rightarrow 0^-} f'(x) = -\infty \end{cases} \quad \text{cusp } x=0$$



Second Derivative

$$f''(x) > 0 \quad 1 < x < 2$$

$$f''(x) < 0 \quad x < 0, 0 < x < 1, x > 2$$

