

1. Does the function  $f(x) = |x|$  on  $[-2, 2]$  satisfy the conditions of the Mean Value Theorem? Why or why not?
2. Determine the vertical and horizontal asymptotes of  $f(x) = \frac{2x^2 + 6x}{x^2 - 9}$
3. Evaluate  $\lim_{x \rightarrow 0} \frac{e^{-5x} - 1 + 5x}{x^2}$
4. Evaluate  $\lim_{x \rightarrow \pi/2} \frac{3 \sec x}{2 + \tan x}$
5. Evaluate  $\lim_{x \rightarrow \pi/2^-} (1 - \sin x) \tan x$
6. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{1}{x^2} - \frac{1}{x^2 \sec x} \right)$
7. Evaluate  $\lim_{x \rightarrow \infty} \left( 1 - \frac{4}{x} \right)^x$
8. Evaluate  $\lim_{x \rightarrow 0^+} (1 + x)^{4/x}$
9. Find the local and absolute extreme values of the function  $f(x) = x - \sqrt{x}$  on  $[0, 4]$ .
10. Given  $f'(x) = (x-1)(x+2)(x+4)$ , determine the critical points of  $f(x)$  and use the second derivative test to determine whether they correspond to local maxima, local minima, or the test is inconclusive.
11. For each function,  $f(x) = (x^2 - 1)^3$  and  $f(x) = x\sqrt{3-x}$ 
  - a. Find the critical points.
  - b. Find intervals of increase and decrease.
  - c. Find local maximum and minimum values.
  - d. Find intervals of concavity and inflection points.
12. For each of the following determine if the statement is True or False; explain your answer.
  - a. If  $f'(c) = 0$  then  $f$  has a maximum or minimum at  $x = c$ .
  - b. If  $f$  is differentiable for all  $x$  in its domain and has an absolute minimum at  $x = c$ , then  $f'(c) = 0$ .
  - c. If  $f$  is continuous on  $(a, b)$  then  $f$  attains an absolute maximum value  $f(c)$  and an absolute minimum value  $f(d)$  at some numbers  $c$  and  $d$  in  $(a, b)$ .
  - d. If  $f''(2) = 0$  then  $(2, f(2))$  is an inflection point of the curve  $y = f(x)$ .
  - e. There exists a function  $f$  such that  $f(1) = -2$ ,  $f(3) = 0$  and  $f'(x) > 1$  for all  $x$ .
13. Sketch the graph of a function that satisfies all the conditions given below.
 
$$f(0) = 0, \lim_{x \rightarrow \pm\infty} f(x) = 2, \lim_{x \rightarrow 4} f(x) = +\infty$$

$$f'(x) < 0 \text{ for } x < 0 \text{ and } x > 4 \text{ and } f'(x) > 0 \text{ for } 0 < x < 4$$

$$f''(x) < 0 \text{ for } x < -2 \text{ and } f''(x) > 0 \text{ for } -2 < x < 4 \text{ and } x > 4$$
14. A closed box with square base is to be built to house an ant colony. The bottom of the box and all four sides are to be made of material costing \$1/ft<sup>2</sup> and the top is to be constructed of glass costing \$5/ft<sup>2</sup>. What are the dimensions of the box of greatest volume that can be constructed for \$72? Verify your answer yields a maximum.
15. Evaluate  $\int \left( 2x^3 - 4x^2 + \frac{4}{x^2} + \frac{1}{x} \right) dx$

16. Evaluate  $\int \frac{x^2 - x + \sqrt{x}}{\sqrt[3]{x}} dx$
17. Evaluate  $\int (\sin 2x + \cos 3x) dx$
18. Evaluate  $\int (\sec^2 \theta + \sec \theta \tan \theta) d\theta$
19. Evaluate  $\int (e^{2x} + e^{-3x}) dx$
20. Evaluate  $\int \frac{1}{16 + x^2} dx$
21. Evaluate  $\int \frac{3}{\sqrt{1 - 4x^2}} dx$
22. Suppose  $\frac{d^2 y}{dx^2} = 7x^2 - 3x$ ,  $y'(1) = 1$  and  $y(1) = 2$ , find  $y$ .

For additional problems, check out the review problems for Chapter 4. Note the questions above are simply a sample of questions possible for the exam; it is possible that other types of questions may appear on your exam.