

1. $\int_{-3}^0 \sqrt{9-x^2} dx = \frac{1}{4}(\text{area of a circle of radius } 3) = \frac{1}{4} \cdot 9\pi \text{ and } \int_0^1 (3-6x) dx = 0 \text{. So}$

$$\int_{-3}^1 f(x) dx = \frac{9\pi}{4}$$

2. $F(x) = \int_{x^2}^1 \frac{1}{\sqrt{1+3t^2}} dt \quad F(x) = - \int_1^{x^2} \frac{1}{\sqrt{1+3t^2}} dt \quad \text{so} \quad F'(x) = -\frac{1}{\sqrt{1+3x^4}} \cdot (2x)$

3.

a. $\int \left(x^2 - \frac{1}{2x} + \frac{1}{x^2} + \cos x \right) dx = \frac{x^3}{3} - \frac{1}{2} \ln|x| - \frac{1}{x} + \sin x + C$

b. $\int (x^3 + 3^x + 3^3) dx = \frac{x^4}{4} + \frac{3^x}{\ln 3} + 3^3 x + C$

c. $\int \frac{-3}{1+16x^2} dx \text{ let } u = 4x \text{ so } 1+16x^2 = 1+u^2 \text{ and } 4dx = du \text{ then}$

$$\int \frac{-3}{1+16x^2} dx = -3 \int \frac{1}{1+u^2} \frac{du}{4} = -\frac{3}{4} \arctan u + C = -\frac{3}{4} \arctan(4x) + C$$

d. $\int \frac{5x^2 - 2x + 1}{\sqrt{x}} dx = \int (5x^{3/2} - 2x^{1/2} + x^{-1/2}) dx = 5 \frac{2}{5} x^{5/2} - 2 \frac{2}{3} x^{3/2} + 2x^{1/2} + C$

e. $\int x^3 e^{3x^4-2} dx \text{ let } u = 3x^4 - 2 \text{ and } du = 12x^3 dx \text{ to get}$

$$\int x^3 e^{3x^4-2} dx = \int \frac{e^u}{12} du = \frac{e^u}{12} + C = \frac{1}{12} e^{3x^4-2} + C$$

f. $\int \frac{6x}{3x^2+1} dx \text{ let } u = 3x^2 + 1 \text{ so } du = 6x dx \text{ to get } \int \frac{6x}{3x^2+1} dx =$

$$\int \frac{du}{u} = \ln|u| + C = \ln|3x^2+1| + C$$

g. $\int \sin(2x+1) dx = -\frac{1}{2} \cos(2x+1) + C$

h. $\int \sec^2(\pi x) dx = \frac{1}{\pi} \tan(\pi x) + C$

i. $\int \frac{1}{4-3x} dx = -\frac{1}{3} \ln|4-3x| + C \quad \text{but} \quad \int_2^4 \frac{1}{4-3x} dx \text{ does not exist since integrand is not continuous.}$

j. $\int_{1/2}^{3/2} 2 \cos(\pi x) dx = \frac{2}{\pi} \sin(\pi x) \Big|_{1/2}^{3/2} = -\frac{4}{\pi}$

k. $\int_0^1 \frac{1}{1+x^2} dx = \arctan x \Big|_0^1 = \frac{\pi}{4}$

l. $\int_{-3}^2 \frac{1}{x+5} dx = \ln(x+5) \Big|_{-3}^2 = \ln 7 - \ln 2$

m. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2e^{\sqrt{x}} + C$

n. $\int_{-1}^1 \frac{2^x}{4^x} dx = \int_{-1}^1 \frac{1}{2^x} dx = \int_{-1}^1 2^{-x} dx = -\frac{1}{\ln 2} 2^{-x} \Big|_{-1}^1 = -\frac{1}{\ln 2} \left(\frac{1}{2} - 2 \right) = \frac{3/2}{\ln 2}$

o. $\int_0^{\pi/4} \tan x \sec^2 x dx = \int_{u=0}^{u=1} u du = \frac{1}{2} \text{ using the substitution } u = \tan x \text{ to get } du = \sec^2 x dx$

4. Let the partition of the interval $[0, 2]$ be $x_0 = 0 < x_1 = \frac{1}{2} < x_2 = \frac{2}{2} < x_3 = \frac{3}{2} < x_4 = \frac{4}{2}$ then a

Riemann sum using midpoints $= \frac{1}{2} \left(\frac{1}{4} \right)^2 + \frac{1}{2} \left(\frac{3}{4} \right)^2 + \frac{1}{2} \left(\frac{5}{4} \right)^2 + \frac{1}{2} \left(\frac{7}{4} \right)^2 = \frac{84}{32} = \frac{21}{8}$

5. The average value is $\frac{1}{2} \int_{-1}^1 e^{2x} dx = \frac{1}{2} \left(\frac{1}{2} e^{2x} \right) \Big|_{-1}^1 = \frac{1}{4} (e^2 - e^{-2})$.

6. The area bounded by $y = x$ and $y = x^2 - 2$ is multiplied by 2 to get

$$2 \int_0^2 (x - (x^2 - 2)) dx = 2 \left(\frac{x^2}{2} - \frac{x^3}{3} + 2x \right) \Big|_0^2 = 2 \left(2 - \frac{8}{3} + 4 \right) = \frac{20}{3}$$

7. Given $4 = \int_4^{12} f(x) dx$ so $\int_1^3 3f(4x) dx = 3 \int_1^3 f(4x) dx = 3 \int_4^{12} f(u) \frac{du}{4} = \frac{3}{4} \int_4^{12} f(u) du = \frac{3}{4}(4) = 3$

where we substitute $u = 4x$

8. $\int_{-2}^2 f(x) dx + \int_2^5 f(x) dx = \int_{-2}^5 f(x) dx$ and $\int_{-2}^{-2} f(x) dx = - \int_{-2}^{-1} f(x) dx$ So we have

$$\int_{-2}^2 f(x) dx + \int_2^5 f(x) dx - \int_{-2}^{-1} f(x) dx = \int_{-2}^2 f(x) dx + \int_2^5 f(x) dx + \int_{-1}^{-2} f(x) dx = \int_{-1}^5 f(x) dx$$

9. $\int_0^4 |\sqrt{x} - 1| dx = \int_0^1 (1 - \sqrt{x}) dx + \int_1^4 (\sqrt{x} - 1) dx = \frac{4}{3} + \frac{5}{3} = 2$

10. Area is $\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} (\sin x - \cos x) dx = (\sqrt{2} - 1) + (\sqrt{2} + 1) = 2\sqrt{2}$