Exam 4 **Name**______________________________

Math 131-01 **Calculus I** **April 23, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 50 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.
- Hint: $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$ and $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$

[8 points] 1. Complete test corrections if score is 84 or below. Test corrections should be completed on a separate sheet of paper. Neatly written, with all work shown. Take the test corrections to the OSL and have them checked by a tutor. The tutor will sign and date the corrections; you will turn these into your instructor. Please DO NOT WAIT till the last minute to have your corrections checked, this may result in your corrections not being done by the due date. Get them done right after the test is returned to you and have them checked early.

2. Evaluate each of the following:

[4 points] a. $\int \left(4\sqrt{x} - \frac{4}{\sqrt{x}}\right) dx$

Solution: $\int \left(4\sqrt{x} - \frac{4}{\sqrt{x}}\right) dx = \int \left(4x^{1/2} - 4x^{-1/2}\right) dx = \frac{8}{3}x^{3/2} - 8x^{1/2} + C$

[4 points] b. $\int \left(e^{4x} + \frac{1}{2x}\right) dx$

Solution: $\int \left(e^{4x} + \frac{1}{2x}\right) dx = \frac{1}{4}e^{4x} + \frac{1}{2}\ln|x| + C$

[4 points] c. $\int (\sin(3x) + \cos(2x))\, dx$

Solution: $\int (\sin(3x) + \cos(2x))\, dx = -\frac{1}{3}\cos(3x) + \frac{1}{2}\sin(2x) + C$

[4 points] d. $\int \frac{x^2 + 2\sqrt[3]{x}}{x} dx$

Solution: $\int \frac{x^2 + 2\sqrt[3]{x}}{x} dx = \int \left(x + 2x^{-2/3}\right) dx = \frac{1}{2}x^2 + 6x^{1/3} + C$

[4 points] e. $\int \frac{1}{1+4x^2} dx$

Solution: $u = 2x$, $du = 2dx$, so $\int \frac{1}{1+4x^2} dx = \frac{1}{2}\int \frac{1}{1+u^2}\, du = \frac{1}{2}\arctan(2x) + C$

[6 points] f. $\int_0^1 \sqrt{x}(\sqrt{x}+1)\, dx$

Solution: $\int_0^1 \sqrt{x}(\sqrt{x}+1)\, dx = \int_0^1 \left(x + x^{1/2}\right) dx = \left(\frac{1}{2}x^2 + \frac{2}{3}x^{3/2}\right)\Big|_0^1 = \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$

[6 points] g. $\int x^2\sqrt{4+3x^3}\, dx$

Solution: $u = 4 + 3x^3$, $du = 9x^2 dx$ so $\frac{1}{9}\int \sqrt{u}\, du = \frac{1}{9}\cdot\frac{2}{3}u^{3/2} = \frac{2}{27}(4+3x^3)^{3/2} + C$

[6 points] h. $\displaystyle\int x^8 \sec^2(x^9)\,dx$

Solution: $u = x^9$, $du = 9x^8dx$, so $\displaystyle\int x^8 \sec^2(x^9)\,dx = \frac{1}{9}\int \sec^2 u\,du = \frac{1}{9}\tan(x^9) + C$

[6 points] i. $\displaystyle\int \frac{1}{10x-3}\,dx$

Solution: Let $u = 10x - 3$, $du = 10dx$, so $\displaystyle\int \frac{1}{10x-3}\,dx = \frac{1}{10}\int \frac{1}{u}\,du = \frac{1}{10}\ln|10x - 3| + C$

[8 points] j. $\displaystyle\int_0^{\ln 4} \frac{e^x}{3+2e^x}\,dx$

Solution: Let $u = 3 + 2e^x$, $du = 2e^x dx$, $u(0) = 3 + 2 = 5$, and $u(\ln 4) = 3 + 2e^{\ln 4} = 3 + 2(4) = 11$ so $\displaystyle\int_0^{\ln 4} \frac{e^x}{3+2e^x}\,dx = \frac{1}{2}\int_5^{11} \frac{1}{u}\,du = \frac{1}{2}\ln|u|\Big|_5^{11} = \frac{1}{2}(\ln 11 - \ln 5) + C$

[8 points] k. $\displaystyle\int_0^{\pi/4} \frac{\sin x}{\cos^3 x}\,dx$

Solution: Let $u = \cos x$, $du = -\sin x dx$, $u(0) = \cos 0 = 1$, and $u(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$, so $\displaystyle\int_0^{\pi/4} \frac{\sin x}{\cos^3 x}\,dx = -\int_1^{\sqrt{2}/2} \frac{1}{u^3}\,du = -\frac{u^{-2}}{-2}\Big|_1^{\sqrt{2}/2} = \frac{1}{2}\left(\left(\frac{2}{\sqrt{2}}\right)^2 - 1\right) = \frac{1}{2}$

[8 points] 3. Find $\displaystyle\frac{d}{dx}\int_x^0 \frac{dp}{p^2+1}$

Solution: $\displaystyle\frac{d}{dx}\int_x^0 \frac{dp}{p^2+1} = -\frac{d}{dx}\int_0^x \frac{dp}{p^2+1} = -\frac{1}{x^2+1}$

4. Suppose $\int_0^4 f(x)\,dx = 10$, and $\int_0^4 g(x)\,dx = 20$. Furthermore, suppose that f is an even function and g is an odd function. Evaluate the following integrals

[4 points] a. Find $\int_{-4}^{4} f(x)\,dx$.

Solution: $\int_{-4}^{4} f(x)\,dx = 2\int_0^4 f(x)dx = 2(10) = 20$

[4 points] b. Find $\int_{-4}^{4} 3g(x)\,dx$

Solution: $\int_{-4}^{4} 3g(x)\,dx = 3\int_{-4}^{4} g(x)\,dx = 0$

[4 points] c. $\int_0^1 8xf(4x^2)\,dx$

Solution: $u = 4x^2$, $du = 8xdx$, so $\int_0^1 8xf(4x^2)\,dx = \int_0^4 f(u)\,du = 10$

4 (bonus)] d. $\int_{-2}^{2} 3xf(x)\,dx$

Solution: Let $F(x) = 3xf(x)$ $F(-x) = 3(-x)f(-x) = -3xf(x) = -F(x)$ so F is an odd function and $\int_{-2}^{2} 3xf(x)\,dx = 0$

5. Given $f(x) = x^2 - x$ on $[0, 3]$

[6 points] a. Find the net area of the region bounded by f and x-axis on the given interval.

Solution: $\int_0^3 \left(x^2 - x\right)\,dx = \left(\frac{1}{3}x^3 - \frac{1}{2}x^2\right)\Big|_0^3 = 9 - \frac{9}{2} = \frac{9}{2}$

[6 points] b. Find the area of the region bounded by f and x-axis on the given interval.

Solution: $\int_0^3 |x^2 - x|\,dx = -\int_0^1 \left(x^2 - x\right)\,dx + \int_1^3 \left(x^2 - x\right)\,dx = -\left(\frac{1}{3}x^3 - \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{3}x^3 - \frac{1}{2}x^2\right)\Big|_1^3 = -\left(\frac{1}{3} - \frac{1}{2}\right) + 9 - \frac{9}{2} - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{1}{6} + \frac{9}{2} + \frac{1}{6} = \frac{29}{6}$

Question	Points	Score
1	8	
2	60	
3	8	
4	12	
5	12	
Total:	100	