Math 131 Review Test 3

For each of the following three equations, find:

1. $f(x) = \dfrac{3x-5}{x-2}$
2. $g(x) = x\sqrt{8-x^2}$
3. . $h(x) = 3x^4 - 4x^2 + 1$
    a. Find where the curve is increasing and where it is decreasing.
    b. Find the local maximums and local minimums if any exist.
    c. Find where the curve is concave up and where it is concave down.
    d. Find the points of inflection if any exist.
    e. Identify any asymptotes.
    f. Plot the graph and label the intercepts and any local maximums and minimums.

4. Evaluate the following:
    a. $\displaystyle\int\left(x^2 - \frac{1}{2x} + \frac{1}{x^2} + \cos x\right)dx$
    b. $\displaystyle\int\left(x^3 + 3^x + 3^3\right)dx$
    c. $\displaystyle\int \frac{-3}{1+16x^2}dx$
    d. $\displaystyle\int \frac{5x^2 - 2x + 1}{\sqrt{x}}dx$

5. Suppose the derivative of a function is $y' = (x-1)^2(x-2)(x-4)$. At what points, if any, does the graph have a local maximum, local minimum, or point of inflection?

6. A function has a domain of $x > 0$, $f'(x) > 0$ when $0 < x < 1$, and $f'(x) < 0$ when $1 < x < \infty$. Can any thing be said about the concavity of $f(x)$?

7. Let $y = ax^2 + bx + c$, where are the local maximum and minimum and what is the concavity?

8. An open-top box is to made by cutting small congruent squares from the corners of a 12-in-by-12-in sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?

9. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a singled strand electric fence. With 800 meters of wire at your disposal, what is the largest area you can enclose with the existing wire?

10. Evaluate the following:
    a. $\displaystyle\lim_{x\to 0}\left(\frac{1-\cos x}{x^2 + x}\right)$
    b. $\displaystyle\lim_{x\to 0^+} x^2 \ln x$
    c. $\displaystyle\lim_{x\to\infty}\left(x - \sqrt{x^2+1}\right)$

d. $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$

11. Graph a function $f(x)$ with the following properties:

$f(-2)=0$ $f(2)=0$

$f'(x)>0$ when $-3<x<0$ or $0<x<3$ and $f'(x)<0$ when $-\infty<x<-3$ or $3<x<\infty$

$f''(x)>0$ when $-5<x<0$ or $5<x<\infty$ and $f''(x)<0$ when $-\infty<x<-5$ or $0<x<5$

$\lim_{x\to 0^+} f(x)=-\infty$ $\lim_{x\to 0^-} f(x)=\infty$ $\lim_{x\to -\infty} f(x)=0$ $\lim_{x\to \infty} f(x)=0$