Exam 3 Version A **Name**______________________________
Math 131-01 **Calculus I** **April 6, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 50 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.

OSL Tutors,
Thank you very much for the thought and care you use when checking solutions for test corrections.

Please sign here: ____

Date: ____________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) Complete test corrections if score is 84 or below. Test corrections should be completed on a separate sheet of paper. Neatly written, with all work shown. Take the test corrections to the OSL and have them checked by a tutor. The tutor will sign and date the corrections; you will turn these into your instructor. Please DO NOT WAIT till the last minute to have your corrections checked, this may result in your corrections not being done by the due date. Get them done right after the test is returned to you and have them checked early.

2. (10 points) Evaluate $\lim_{x \to e} \dfrac{\ln x - 1}{x - e}$.

Solution:
$\lim_{x \to e} \dfrac{\ln x - 1}{x - e} = \lim_{x \to e} \dfrac{1/x}{1} = \dfrac{1/e}{1} = \dfrac{1}{e}$.

Note this problem is supposed to be straight forward and hopefully students earn many points. If they only write something like 0/0 showing they realize it is indeterminate, 2 points. Otherwise 7 points for using L'hopital and 3 points for the correct answer.

3. (10 points) Evaluate $\lim_{x \to 0} \dfrac{\sin^2(3x)}{3x^2}$.

Solution:
$\lim_{x \to 0} \dfrac{\sin^2(3x)}{3x^2} = \lim_{x \to 0} \dfrac{6\cos(3x)\sin(3x)}{6x} = \lim_{x \to 0} \dfrac{18\cos^2(3x) - 18\sin^2(3x)}{4} = \dfrac{18}{6} = 3$
Note 5 points for the first derivative limit of the L'hop. then 5 points for the second limit and answer.
If they get the derivative wrong on the first step, say they forget the chain rule, then they are going to get then next part wrong. However, if they do everything right continuing their chain rule problem, you can give them 5 points.

4. (10 points) Evaluate $\lim_{x \to \infty} \left(1 - \dfrac{12}{x^2}\right)^{x^2}$.

Solution:
$\lim_{x \to \infty} \left(1 - \dfrac{12}{x^2}\right)^{x^2} = \lim_{x \to \infty} e^{\ln\left(1 - \frac{12}{x^2}\right)^{x^2}} = \exp\left(\lim_{x \to \infty} \dfrac{\ln(1 - 12/x^2)}{1/x^2}\right)$
$= \exp\left(\lim_{x \to \infty} \dfrac{\frac{1}{1-12/x^2} \cdot 24/x^3}{-2/x^3}\right) = e^{-12}$.
Note: Grade like previous. They need to show they to use the natural log, 3 points. Then the derivative of the limit and limit -12, 4 points, then the limit is a power of the exponential function 3 points.

5. (10 points) Approximate the change in the volume of a sphere when its radius changes from $r = 4$ ft to $r = 4.1$ ft $\left(V(r) = \dfrac{4}{3}\pi r^3\right)$.

Solution:

$V'(r) = 4\pi r^2$, $V'(4) = 64\pi$ so $\Delta V \approx 64\pi(r-4)$ and for our change in r, $\Delta V = 64\pi/10 = 6.4\pi$.

Note: Find the derivative of V, 2 points. Evaluate derivative at $r = 4$, 2 points. Formula for approximate change in V, 4 points, then show the change in V is approximately V'(4)*(4.1-4), 2 points

6. (10 points) Given $f(x) = x^2 - 8\ln x$, find intervals of increase and decrease for the function. Label each critical point as local maximum, local minimum or neither.

> **Solution:**
> $f'(x) = 2x - \dfrac{8}{x} = \dfrac{2(x^2-4)}{x}$, thus there are CP at $x = 0$ and $x = \pm 2$
> However since the domain of the function is $(0, \infty)$ CP at $x = 2$
>
> Increasing on $(2, \infty)$
> Decreasing on $(0, 2)$
> Local minimum at $x = 2$
>
> Note: Find the derivative, 4 points; they realize that domain is (0, infinity) so only cp is x=2, 2 points. State interval of increase, 1 points, decrease 1 points, 2 point stating x=2 is local minimum.
>
> Note if they miss the derivative, grade their work based on their derivative.

7. (10 points) Suppose $f''(x) = \dfrac{12x^4 - 12x}{(2+x^3)^3}$ for a function with domain $(-\infty, -\sqrt[3]{2}) \cup (-\sqrt[3]{2}, \infty)$. Determine intervals where the function is concave up and concave down. Also identify any points of inflection.

> **Solution:**
> Concave down $(-\infty, -\sqrt[3]{2})$, $(0, 1)$ Concave up on $(-\sqrt[3]{2}, 0)$, $(1, \infty)$
> Points of inflection at $x = 0$ and $x = 1$
> Note: Find critical points, 2 points, 2 points for intervals of concave up and 2 points for intervals of concave down, 2 points for inflection points. If everything else is correct but they claim $x = -\sqrt[3]{2}$ is a point of inflection, take off 3 points.

8. (10 points) Find the absolute maximum and minimum for $g(x) = x^3 e^{-x}$ on $[-1, 5]$

> **Solution:**
> $f'(x) = 3x^2e^{-x} - x^3e^{-x} = x^2e^{-x}(3-x)$ there are critical points at $x = 0$ and $x = 3$. $f(-1) = -e^1$, $f(0) = 0$, $f(3) = 27 \cdot e^{-3}$, and $f(5) = 125 \cdot e^{-5}$ Thus the absolute maximum occurs at $(3, 27/e^3)$ and the absolute minimum is at $(-1, -e)$.
>
> Note: Find the derivative and determine the CPs, 4 points, then evaluate the function at the endpoints, 2 points, and the cps, 2 points. State conclusion, 2 points

9. (10 points) Sketch a graph of a function that satisfies the following conditions:

   a. $f(0) = 0$, $\lim_{x \to \pm\infty} f(x) = 4$, and $\lim_{x \to 2^+} f(x) = \infty$, $\lim_{x \to 2^-} f(x) = -\infty$

   b. $f'(x) < 0$ for $-\infty < x < 0$, $0 < x < 2$, $2 < x < \infty$

   c. $f''(x) < 0$ for $-\infty < x < -2$ and $0 < x < 2$; $f''(x) > 0$ for $-2 < x < 0$ and $2 < x < \infty$

Solution:

Graph needs to go through the origin, 1 point, have horizontal asymptote at $y = 4$, 2 points, a vertical asymptote at $x = 2$, 2 points, it should always be decreasing, 2 points, show there is a point of inflection at $x = -2$ and correct concavity on the various intervals, 3 points.

10. (10 points) A square based box-shaped shipping crate is designed to have a volume of 16 cubic feet. The material used to make the base costs $8 per square foot, the material to make the sides cost $4 per square foot and $2 per square foot for the top. What are the dimensions of the crate that minimizes the cost of the materials?

Solution:

$V = 16 = x^2 y$ which implies that $y = 16/x^2$.
Cost: $C = 8x^2 + 16xy + 2x^2$ with the constraint $C(x) = 10x^2 + 16^2/x$ for $x > 0$.
So $C' = 20x - 256/x^2 = 0$ at $x^3 = 256/20 = 64/5$ and $x = 4/\sqrt[3]{5}$.
$C'' = 20 + 512/x^3 > 0$ for all $x > 0$. Thus cost is minimized at $x = 4/\sqrt[3]{5}$ and $y = 5^{2/3}$

Notes: Find the function for cost, C(x), 3 points. Find the derivative of cost, 3 points and find critical point 4 points.
You can give partial credit for pictures, say 2 points to give them a little something.