

Determine the derivative for each of the following:

$$1. \quad y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}} \quad \frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3}$$

$$2. \quad y = e^{-2x} \quad \frac{dy}{dx} = -2e^{-2x}$$

$$3. \quad y = 7^{x^2} = e^{x^2 \ln 7} \quad \frac{dy}{dx} = e^{x^2 \ln 7} (2x \ln 7)$$

$$4. \quad y = \sec(2x) \quad \frac{dy}{dx} = 2\sec(2x)\tan(2x)$$

$$5. \quad y = \sinh(x^2) \quad \frac{dy}{dx} = 2x \cosh(x^2)$$

$$6. \quad y = \ln(\pi x) \quad \frac{dy}{dx} = \frac{1}{\pi x}(\pi)$$

$$7. \quad y = \tan(1-2x) \quad \frac{dy}{dx} = -2\sec^2(1-2x)$$

$$8. \quad y = \ln 2 \quad \frac{dy}{dx} = 0$$

$$9. \quad y = \cos^3 x \quad \frac{dy}{dx} = 3\cos^2 x(-\sin x)$$

$$10. \quad y = \cot(3x^2) \quad \frac{dy}{dx} = -6x \csc^2 x$$

$$11. \quad y = \log_3 x^2 = \frac{\ln x^2}{\ln 3} \quad \frac{dy}{dx} = \frac{1}{\ln 3} \left(\frac{1}{x^2} 2x \right)$$

$$12. \quad y = \frac{1}{7x+3} = (7x+3)^{-1} \quad \frac{dy}{dx} = -(7x+3)^{-2}(7)$$

$$13. \quad y = \arcsin(4x^2) \quad \frac{dy}{dx} = \frac{1}{\sqrt{1-(4x^2)^2}}(8x)$$

$$14. \quad y = \left(9x + \frac{2}{x}\right)^4 \quad \frac{dy}{dx} = 4\left(9x + \frac{2}{x}\right)^3 \left(9 - \frac{2}{x^2}\right)$$

$$15. \quad y = \arctan(\sqrt{x}) \quad \frac{dy}{dx} = \frac{1}{1+(\sqrt{x})^2} \left(\frac{1}{2\sqrt{x}} \right)$$

$$16. \quad y = e^{\sin x} \quad \frac{dy}{dx} = (\cos x)e^{\sin x}$$

$$17. \quad y = \ln(x^2 + 2x) \quad \frac{dy}{dx} = \frac{2x+2}{x^2 + 2x}$$

$$18. \quad y = \sin(e^x + x) \quad \frac{dy}{dx} = \cos(e^x + x) \cdot (e^x + 1)$$

$$19. \quad y = (x + x^{-1})^4 \quad \frac{dy}{dx} = 4(x + x^{-1})^3(1 - x^{-2})$$

$$20. \quad y = \sqrt[3]{3x^7 + 4x^3 + 3} \quad \frac{dy}{dx} = \frac{1}{3}(3x^7 + 4x^3 + 3)^{-2/3}(21x^6 + 12x^2)$$

Find the derivative, $f'(x)$ for each of 21-29

$$21. \quad f(x) = (x^3 + \cosh x)(x + \ln x)$$

$$f'(x) = (3x^2 + \sinh x)(x + \ln x) + (x^3 + \cosh x)\left(1 + \frac{1}{x}\right)$$

$$22. \quad f(x) = \frac{\tan x}{x^2 + 1} \quad f'(x) = \frac{(\sec^2 x)(x^2 + 1) - (\tan x)2x}{(x^2 + 1)^2}$$

$$23. \quad f(x) = \sin^3(\ln(x+3)) \quad f'(x) = 3\sin^2(\ln(x+3)) \cdot \frac{1}{x+3}$$

$$24. \quad f(x) = x^3 + x^{\sqrt{3}} - x^{-2} \quad f'(x) = 3x^2 + \sqrt{3}x^{\sqrt{3}-1} + 2x^{-3}$$

$$25. \quad f(x) = e^{2x} \arcsin(x^2 + 1) \quad f'(x) = 2e^{2x} \arcsin(x^2 + 1) + e^{2x} \frac{2x}{\sqrt{1-(x^2+1)^2}}$$

$$26. \quad f(x) = x \sin(3x+1) \quad f'(x) = \sin(3x+1) + 3x \cos(3x+1)$$

$$27. \quad f(x) = x^{\tan x}$$

$$\ln y = \ln x^{\tan x} = \tan x(\ln x)$$

$$\frac{1}{y} \frac{dy}{dx} = \sec^2 x(\ln x) + (\tan x)\left(\frac{1}{x}\right) \text{ so } \frac{dy}{dx} = x^{\tan x} \left[\sec^2 x(\ln x) + (\tan x)\left(\frac{1}{x}\right) \right]$$

$$28. \quad f(x) = \frac{(2x+1)^2(3x+2)^5(x+3)}{7x(5x-1)^3}$$

$$\ln y = 2\ln(2x+1) + 5\ln(3x+2) + \ln(x+3) - \ln(7x) - 3\ln(5x-1)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{4}{2x+1} + \frac{15}{3x+2} + \frac{1}{x+3} - \frac{1}{x} - \frac{15}{5x-1} \text{ so } \frac{dy}{dx} = y \left[\frac{4}{2x+1} + \frac{15}{3x+2} + \frac{1}{x+3} - \frac{1}{x} - \frac{15}{5x-1} \right]$$

$$29. \quad f(x) = \tan^{-1}(\sin(x^2))$$

$$30. \quad \text{Find } \frac{dy}{dx} \text{ and } \frac{d^2y}{dx^2} \text{ where } y = \frac{\sin x}{x^2 + 1}$$

$$\frac{dy}{dx} = \frac{\cos x(x^2 + 1) - \sin x(2x)}{(x^2 + 1)}, \quad \frac{d^2y}{dx^2} = \frac{-\sin x(x^2 + 1) + \cos x(2x) - \cos x(2x) - 2\sin x}{(x^2 + 1)^4}$$

$$31. \quad \text{If } y = e^{3x} \text{ then find } f'(x), f''(x), f'''(x), f^{(4)}(x).$$

$$f'(x) = 3e^{3x}, f''(x) = 9e^{3x}, f'''(x) = 27e^{3x}, f^{(4)}(x) = 81e^{3x}$$

32. Give an equation of the tangent line to $xy^2 + 2x^2y = 8$ at the point (1,2).

$$m_{\tan} = \left. \frac{dy}{dx} \right|_{(1,2)} = -\left. \frac{y^2 + 4xy}{2xy + 2x^2} \right|_{(1,2)} = -2 \text{ so a tangent line is } y - 2 = -2(x - 1)$$

33. Use the definition of derivative to find the derivative of $y = x^2 + 2x$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 2h}{h} = \lim_{h \rightarrow 0} \frac{h(2x + h + 2)}{h} \\ &= \lim_{h \rightarrow 0} (2x + h + 2) = 2x + 2 \end{aligned}$$

34. Use the definition of the derivative to find $f'(1)$ where $f(x) = \sqrt{3+x}$.

$$\begin{aligned} f'(1) &= \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - \sqrt{4}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2} \\ &= \lim_{h \rightarrow 0} \frac{4+h-4}{h(\sqrt{4+h}+2)} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{4} \end{aligned}$$

35. Find an equation of the tangent line to $y = \sin^3(x)$ at $x = \frac{\pi}{4}$.

$$f'(x) = 3\sin^2 x \cos x$$

$$y_0 = f\left(\frac{\pi}{4}\right) = \left(\sin\left(\frac{\pi}{4}\right)\right)^3 = \frac{\sqrt{2}}{4} \text{ and } m_{\tan} = f'\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{4}$$

$$\text{So a tan line is } y - \frac{\sqrt{2}}{4} = \frac{3\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right).$$