Exam 2 Version A **Name** Solution

Math 131-01 **Calculus I** **March 6, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 50 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers or on the line provided.
- DO NOT SIMPLIFY YOUR DERIVATIVES
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write see other side and continue working on the back of the same page.

Question	Points	Score
1	8	
2	26	
3	6	
4	6	
5	6	
6	8	
7	8	
8	10	
9	12	
10	10	
Total:	100	

1. (8 points) Complete test corrections if score is 84 or below. Test corrections should be completed on a separate sheet of paper. Neatly written, with all work shown. Take the test corrections to the OSL and have them checked by a tutor. The tutor will sign and date the corrections; you will turn these into your instructor. Please DO NOT WAIT till the last minute to have your corrections checked, this may result in your corrections not being done by the due date. Get them done right after the test is returned to you and have them checked early. t

2. Find $\frac{dy}{dx}$ for each of the following:

a. (2 points) $y = \pi x^2 + \frac{5}{2x^2} = \pi x^2 + \frac{5}{2} x^{-2}$

b. (2 points) $y = \pi^3$

c. (2 points) $y = \ln(2x^8)$

d. (2 points) $y = (6x^4 + 2x^3)^{62}$

e. (2 points) $y = \sin(5x)$

f. (2 points) $y = e^{2/x^3}$

g. (2 points) $y = \tan^3 x$

h. (2 points) $\sqrt[4]{9 + 7x}$

i. (2 points) $y = \log_7(x^2) = \frac{\ln(x^2)}{\ln 7}$

j. (2 points) $y = \arctan \sqrt[3]{x}$

k. (2 points) $y = 3^{\cos x} = e^{\cos x \cdot \ln 3}$

l. (2 points) $y = \arcsin(3x)$

m. (2 points) $y = \frac{7}{x^5 + 8x^2} = 7(x^5+8x^2)^{-1}$

a. $y' = 2\pi x - 5x^{-3}$

b. $y' = 0$

c. $y' = \frac{1}{2x^8} \cdot 16x^7 = \frac{8}{x}$

d. $y' = 62(6x^4+2x^3)^{61}(24x^3+6x)$

e. $y' = 5\cos(5x)$

f. $e^{2/x^3} \cdot -6x^{-4}$

g. $3\tan^2 x \cdot \sec^2 x$

h. $\frac{1}{4}(9+7x)^{-3/4}(7)$

i. $y' = \frac{1}{\ln 7} \cdot \frac{2}{x}$

j. $y' = \frac{1}{1+x^{2/3}} \cdot \frac{1}{3} x^{-2/3}$

k. $-\sin x \cdot \ln 3 \, e^{\cos x \ln 3}$

l. $\frac{1}{\sqrt{1-9x^2}} \cdot 3$

m. $-7(x^5+8x^2)^{-2}(5x^4+16x)$

3. (6 points) Find dy/dx for $y = e^{6x}\tan(3x)$

$$\frac{dy}{dx} = 6e^{6x}\tan(3x) + 3e^{6x}\sec^2(3x)$$

4. (6 points) Find dy/dx for $y = \dfrac{x^3 + 2x}{4 - 3x}$

$$\frac{dy}{dx} = \frac{(3x^2+2)(4-3x) - (x^3+2x)(-3)}{(4-3x)^2}$$

5. (6 points) Find dy/dx for $y = x^2 + 3x$ using the definition of derivative.

$$y' = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{1}{h}\left[(x+h)^2 + 3(x+h) - x^2 - 3x\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h \cancel{-x^2} \cancel{-3x}\right]$$

$$= \lim_{h\to 0}\frac{1}{\cancel{h}}\left[\cancel{h}(2x+h+3)\right] = \underline{2x+3}$$

6. (8 points) Find dy/dx for $7x^3 + 5y^3 = 11xy$.

$$21x^2 + 15y^2 \cdot \frac{dy}{dx} = 11y + 11x\frac{dy}{dx}$$

$$(15y^2 - 11x)\frac{dy}{dx} = 11y - 21x^2$$

$$\boxed{\frac{dy}{dx} = \frac{11y - 21x^2}{15y^2 - 11x}}$$

7. (8 points) Find dy/dx for $y = \left(1 + \frac{1}{x}\right)^{2x} = e^{2x\ln\left(1+\frac{1}{x}\right)}$

$$\boxed{y' = e^{2x\ln\left(1+\frac{1}{x}\right)}\left[2\ln\left(1+\frac{1}{x}\right) + 2x \cdot \frac{1}{\left(1+\frac{1}{x}\right)} \cdot \frac{-1}{x^2}\right]}$$

might see $\ln y = 2x\ln\left(1+\frac{1}{x}\right)$

$$\frac{1}{y}\frac{dy}{dx} = 2\ln\left(1+\frac{1}{x}\right) + 2x \cdot \frac{1}{1+1/x} \cdot \frac{-1}{x^2}$$

$$\frac{dy}{dx} = y\left[2\ln\left(1+\frac{1}{x}\right) - \frac{2}{x} \cdot \frac{1}{1+1/x}\right]$$

8. (10 points) Find an equation of the line tangent to the curve $y = x\sqrt{8 - x^2}$ at $x = 2$.

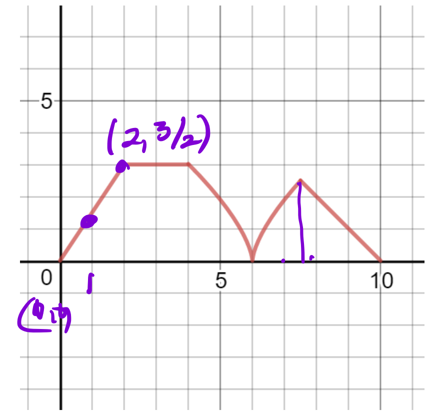
$$y_0 = 2\sqrt{8-4} = 2\sqrt{4} = 4$$

$$y' = \sqrt{8-x^2} + x \cdot (8-x^2)^{-1/2} \cdot -2x = \sqrt{8-x^2} - \frac{2x^2}{\sqrt{8-x^2}}$$

$$m = \sqrt{8-4} - \frac{2(4)}{\sqrt{4}} = 2 - \frac{8}{2} = -2$$

$$\boxed{y - 4 = -2(x-2)} \text{ or } \boxed{y = -2x}$$

9. Given the position function, $s = f(t)$, seen below, answer the following questions:



$m = \frac{3/2}{2} = 3/4$

a. (3 points) $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

b. (3 points) Where is $\frac{ds}{dt} < 0$?

c. (3 points) Where is $f'(t) = 0$?

d. (3 points) Where is $f'(t)$ undefined?

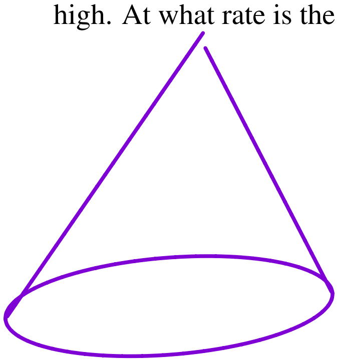
a. $3/4$

b. $4 < x < 6, \quad 7.5 < x < 10$

c. $2 < x < 4$

d. $x = 2, 4, 5, 7.5$

10. (10 points) Sand falls from an overhead bin and accumulates in a conical pile with a radius that is always three times its height. Suppose the height of the pile increases at a rate of 2 cm/s when the pile is 12 cm high. At what rate is the sand leaving the bin at that instant? Hint: $V = \frac{1}{3}\pi r^2 h$.



find $\frac{dV}{dt}$ $\quad r = 3h \quad \frac{dh}{dt} = \frac{2\text{cm}}{s} \quad h = 12\text{cm}$

$V = \frac{1}{3}\pi (3h)^2 h = 3\pi h^3$

$\frac{dV}{dt} = 9\pi h^2 \frac{dh}{dt}$

change of pile

$\frac{dV}{dt} = 9\pi (12)^2 (2) = 18\pi (144)\ \text{cm}^3/s$

$\rightarrow$ $\boxed{\text{leaving bin } \frac{dV}{dt} = -18\pi (144)\text{cm}^3/s}$