Exam 1 Name Solution

Math 131-01 **Calculus I** **February 5, 2018**

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 50 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.

Limit Laws

Suppose that k is a constant, and the limits $\lim\limits_{x\to c} f(x)$ and $\lim\limits_{x\to c} g(x)$ exist.

1. Constant Multiple: $\lim\limits_{x\to c} k\, f(x) = k \lim\limits_{x\to c} f(x)$
2. Sum/Difference Rule: $\lim\limits_{x\to c} \Big(f(x) \pm g(x)\Big) = \lim\limits_{x\to c} f(x) \pm \lim\limits_{x\to c} g(x)$
3. Product Rule: $\lim\limits_{x\to c} \Big(f(x) \cdot g(x)\Big) = \lim\limits_{x\to c} f(x) \cdot \lim\limits_{x\to c} g(x)$
4. Quotient Rule: $\lim\limits_{x\to c} \left(\dfrac{f(x)}{g(x)}\right) = \dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$ if $\lim\limits_{x\to c} g(x) \neq 0$
5. Composition Rule: $\lim\limits_{x\to c} f\big(g(x)\big) = f\big(\lim\limits_{x\to c} g(x)\big)$, if f is continuous at $\lim\limits_{x\to c} g(x)$
6. Cancellation Theorem for Limits: If $\lim\limits_{x\to c} g(x)$ exists and f is a function that is equal to g for all x sufficiently close to c, except possibly at c itself, then $\lim\limits_{x\to c} f(x) = \lim\limits_{x\to c} g(x)$.
7. Squeeze Theorem for Limits: If $l(x) \leq f(x) \leq u(x)$ for all x sufficiently close to c, but not necessarily at $x = c$, and if $\lim\limits_{x\to c} l(x) = L = \lim\limits_{x\to c} u(x)$, then $\lim\limits_{x\to c} f(x) = L$.
8. Limits Whose Denominators Approach Zero from the Right or the Left:
   (a) If $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)}$ is of the form $\dfrac{1}{0^+}$, then $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \infty$.
   (b) If $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)}$ is of the form $\dfrac{1}{0^-}$, then $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = -\infty$.
9. Limits Whose Denominators Become Infinite Approach Zero:
   (a) If $\lim\limits_{x\to \infty} \dfrac{f(x)}{g(x)}$ is of the form $\dfrac{1}{\infty}$, then $\lim\limits_{x\to \infty} \dfrac{f(x)}{g(x)} = 0$.
   (b) If $\lim\limits_{x\to \infty} \dfrac{f(x)}{g(x)}$ is of the form $\dfrac{1}{-\infty}$, then $\lim\limits_{x\to \infty} \dfrac{f(x)}{g(x)} = 0$.
10. $\lim\limits_{\theta\to 0} \dfrac{\sin(\theta)}{\theta} = 1, \qquad \lim\limits_{\theta\to 0} \dfrac{1-\cos(\theta)}{\theta} = 0, \qquad \lim\limits_{x\to 0} (1+x)^{1/x} = e.$

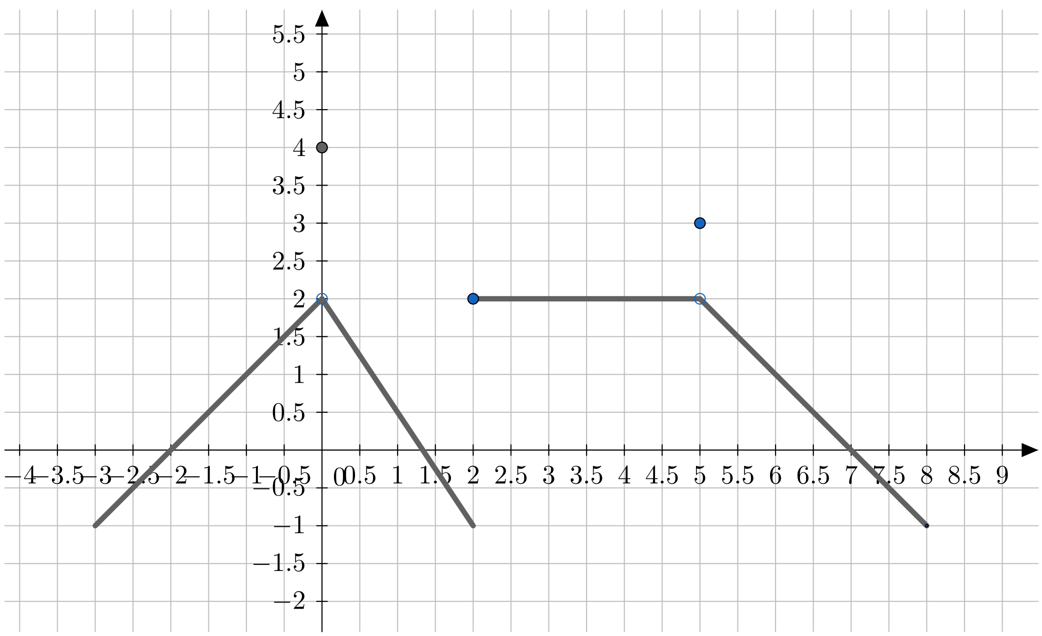
Similar results hold for limits at infinity and one-sided limits.

1. (6 points) Write an equation of a line through the point $(1, 3)$ and perpendicular to the line through $(1, 3)$ and $(-1, -3)$.

$m = \frac{-3-3}{-1-1} = \frac{-6}{-2} = 3$ So slope $\perp$ line is $m_{\perp} = -\frac{1}{3}$

$\boxed{\text{Line } y - 1 = -\frac{1}{3}(x-1) \text{ or } y = -\frac{1}{3}x + \frac{10}{3}}$

2. (20 points) Use the function $f(x)$ below and limit laws (see cover page, show work using the rules where appropriate) to find the following limits, or explain why they do not exist.



4pt a. $\lim_{x \to 0} f(x) = 2$

2pt b. $\lim_{x \to 2^+} f(x) = 2$

2pt c. $\lim_{x \to 2^-} f(x) = -1$

4 pt d. $\lim_{x\to 2} f(x) = DNE$ bc $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

4 pt e. $\lim_{x\to 5} \sqrt[3]{4f(x)} \overset{(5,2)}{=} \sqrt[3]{4\lim_{x\to 5} f(x)} = \sqrt[3]{8} = 2$

4 pt f. $\lim_{x\to 2^-} \dfrac{2f(x)+3}{f(x)-9} \underset{\text{rules } 1,2,4}{=} \dfrac{2\lim f(x) + \lim 3}{\lim_{x\to 2^-} f(x) - \lim_{x\to 2^-} 9} = \dfrac{2(-1)+3}{-1-9} = -\dfrac{1}{10}$

3. Evaluate the following limits, if they exist (show your work).

a. (6 points) $\lim_{x\to 3^+} \dfrac{x^2-7x+12}{x^2-5x+6} = \lim_{x\to 3^+} \dfrac{\cancel{(x-3)}(x-4)}{\cancel{(x-3)}(x-2)} = \dfrac{-1}{1} = \boxed{-1}$

b. (6 points) $\lim_{x\to 0} \dfrac{\frac{1}{4-x}-\frac{1}{4}}{x} = \lim_{x\to 0} \dfrac{1}{x}\left[\dfrac{4}{4(4-x)} - \dfrac{4-x}{4(4-x)}\right]$

$= \lim_{x\to 0} \dfrac{1}{\cancel{x}}\left[\dfrac{1\cancel{x}}{4(4-x)}\right] = \boxed{\dfrac{1}{16}}$

c. (6 points) $\lim_{x\to 1} \dfrac{\sqrt{2x+7}-3}{x-1} \cdot \dfrac{\sqrt{2x+7}+3}{\sqrt{2x+7}+3} = \lim \dfrac{\overset{2(x-1)}{2x-2}}{(x-1)\left[\sqrt{2x+7}+3\right]}$

$= \lim_{x\to 1} \dfrac{2}{\sqrt{2x+7}+3} = \dfrac{2}{6} = \boxed{\dfrac{1}{3}}$

d. (6 points) $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} x & \text{if } x<2 \\ 4 & \text{if } x=2 \\ 6-2x & \text{if } x>2 \end{cases}$

$\lim_{x\to 2^-} x = 2, \quad \lim_{x\to 2^+} (6-2x) = 2 \Rightarrow \boxed{\lim_{x\to 2} f(x) = 2}$

e. (6 points) $\lim_{x\to 4^-} \dfrac{|x-4|}{x^2-3x-4} = \lim_{x\to 4^-} \dfrac{-\cancel{(x-4)}}{\cancel{(x-4)}(x+1)} = \boxed{\dfrac{-1}{5}}$

f. (6 points) $\lim_{x\to\infty} \dfrac{2x^3-2x^2+x}{\sqrt{9x^6+4x^3+x+3}} \approx \lim_{x\to\infty} \dfrac{2x^3}{3x^3} = \boxed{\dfrac{2}{3}}$

g. (6 points) $\lim_{x \to -2} f(x)$, where $f(x) = \begin{cases} x^2 & \text{if } x \geq -2 \\ 7 - x & \text{if } x < -2 \end{cases}$

$\lim_{x \to 2^-}(7-x) = 9 \neq \lim_{x \to -2^+}(x^2) = 4$ $\therefore$ $\boxed{\lim_{x \to 2} f(x) \text{ DNE}}$

4. (6 points) If $P(x) = 2x^3 - 5x^2 - 10x + 5$, use the Intermediate Value Theorem to show that $P(x)$ has a root somewhere in the interval $[-1, 2]$.

$P(x)$ is a polynomial $\therefore$ continuous on $[-1,2]$

$P(-1) = 8$, $P(2) = -19$ $\rightarrow$ $P(2) < 0 < P(-1)$

$\therefore$ there is a $c \in (-1,2)$ where $P(c) = 0$ by IVT

5. (8 points) Let

$$f(x) = \begin{cases} x^2 - b & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ ax + 4 & \text{if } x > 1 \end{cases}$$

Find values of a and b such that f is continuous at $x = \cancel{2}$ 1. Use the definition of continuity in your answer.

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$f(1) = 2$ $\quad \lim_{x \to 1^-}(x^2 - b) = 1 - b$ $\quad \lim_{x \to 1^+}(ax + 4) = a + 4$

So $1 - b = 2$ & $a + 4 = 2$

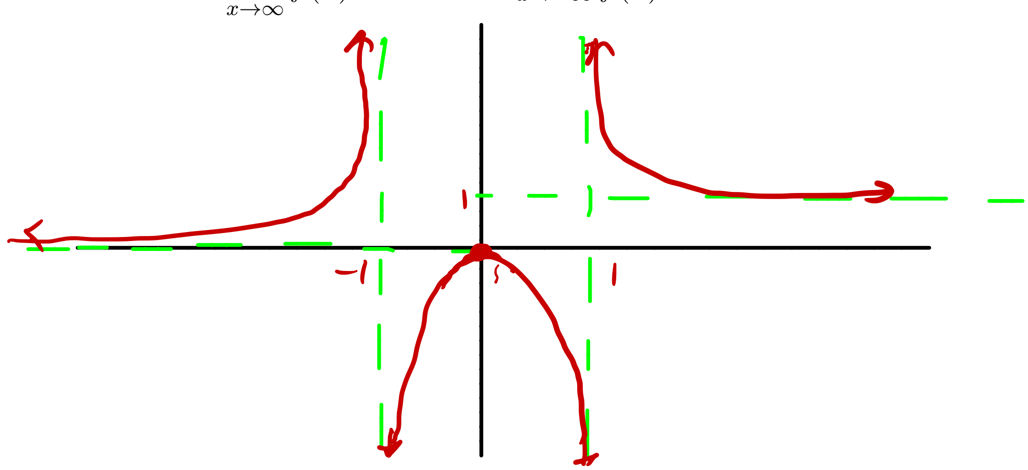
$\boxed{b = -1 \qquad a = -2}$

6. (10 points) Sketch a graph of a function f that satisfies all the following conditions.

$\lim_{x \to 1^+} f(x) = \infty \quad \lim_{x \to 1^-} f(x) = -\infty \quad f(0) = 0$

$\lim_{x \to -1^-} f(x) = \infty \quad \lim_{x \to -1^+} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = 1 \quad \lim_{x \to -\infty} f(x) = 0$



7. (8 points) To be done when exam is graded and returned. On a separate sheet of paper, correct ALL missed problems if your score is less than 85 points. Then take the corrections and exam to the OSL, a tutor will check your work. They will also explain questions you are having difficulty correcting.

Question	Points	Score
1	6	
2	20	
3	42	
4	6	
5	8	
6	10	
7	8	
Total:	100	